

1.11 The vertical displacement of a string is given by the harmonic function

$$y(x, t) = 2 \cos(16\pi t - 20\pi x) \quad (\text{m}),$$

where x is the horizontal distance along the string in meters. Suppose a tiny particle were attached to the string at $x = 5$ cm. Obtain an expression for the vertical velocity of the particle as a function of time.

Solution:

$$y(x, t) = 2 \cos(16\pi t - 20\pi x) \quad (\text{m}).$$

$$\begin{aligned} u(0.05, t) &= \left. \frac{dy(x, t)}{dt} \right|_{x=0.05} \\ &= -32\pi \sin(16\pi t - 20\pi x)|_{x=0.05} \\ &= -32\pi \sin(16\pi t - \pi) \\ &= 32\pi \sin(16\pi t) \quad (\text{m/s}). \end{aligned}$$
