2.26 A 50-Ω lossless transmission line is connected to a load composed of a 75-Ω resistor in series with a capacitor of unknown capacitance (Fig. P2.26). If at 10 MHz the voltage standing wave ratio on the line was measured to be 3, determine the capacitance C.

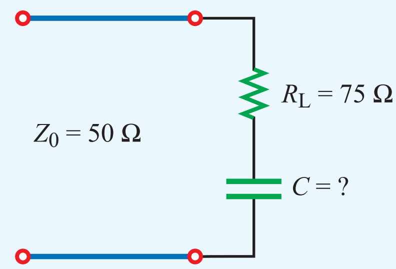

Figure P2.26 Circuit for Problem 2.26.

Solution:

$$|\Gamma| = \frac{S-1}{S+1} = \frac{3-1}{3+1} = \frac{2}{4} = 0.5$$

$$Z_L = R_L - jX_C, \qquad \text{where } X_C = \frac{1}{\omega C}.$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$|\Gamma|^2 = \left[\left(\frac{Z_L - Z_0}{Z_L + Z_0}\right)\left(\frac{Z_L^* - Z_0}{Z_L^* + Z_0}\right)\right]$$

$$|\Gamma|^2 = \frac{Z_L Z_L^* + Z_0^2 - Z_0(Z_L + Z_L^*)}{Z_L Z_L^* + Z_0^2 + Z_0(Z_L + Z_L^*)}$$

Noting that:

$$Z_L Z_L^* = (R_L - jX_C)(R_L + jX_C) = R_L^2 + X_C^2,$$
$$Z_0(Z_L + Z_L^*) = Z_0(R_L - jX_C + R_L + jX_C) = 2Z_0 R_L,$$
$$|\Gamma|^2 = \frac{R_L^2 + X_C^2 + Z_0^2 - 2Z_0 R_L}{R_L^2 + X_C^2 + Z_0^2 + 2Z_0 R_L}.$$

Upon substituting $|\Gamma_L| = 0.5$, $R_L = 75\ \Omega$, and $Z_0 = 50\ \Omega$, and then solving for X_C, we have

$$X_C = 66.1\ \Omega.$$

Hence

$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi \times 10^7 \times 66.1} = 2.41 \times 10^{-10} = 241\ \text{pF}.$$