2.27 At an operating frequency of 300 MHz, a lossless 50-Ω air-spaced transmission line 2.5 m in length is terminated with an impedance $Z_L = (40 + j20)\ \Omega$. Find the input impedance.

Solution: Given a lossless transmission line, $Z_0 = 50\ \Omega$, $f = 300$ MHz, $l = 2.5$ m, and $Z_L = (40 + j20)\ \Omega$. Since the line is air filled, $u_p = c$ and therefore, from Eq. (2.48),

$$\beta = \frac{\omega}{u_p} = \frac{2\pi \times 300 \times 10^6}{3 \times 10^8} = 2\pi \text{ rad/m}.$$

Since the line is lossless, Eq. (2.79) is valid:

$$\begin{aligned} Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) &= 50 \left[\frac{(40 + j20) + j50 \tan (2\pi \text{ rad/m} \times 2.5 \text{ m})}{50 + j(40 + j20) \tan (2\pi \text{ rad/m} \times 2.5 \text{ m})} \right] \\ &= 50 \left[(40 + j20) + j50 \times 0 \right] 50 + j(40 + j20) \times 0 \\ &= (40 + j20)\ \Omega. \end{aligned}$$