

2.31 A voltage generator with

$$v_g(t) = 5 \cos(2\pi \times 10^9 t) \text{ V}$$

and internal impedance $Z_g = 50 \Omega$ is connected to a $50\text{-}\Omega$ lossless air-spaced transmission line. The line length is 5 cm and the line is terminated in a load with impedance $Z_L = (100 - j100) \Omega$. Determine:

- (a) Γ at the load.
- (b) Z_{in} at the input to the transmission line.
- (c) The input voltage \tilde{V}_i and input current \tilde{I}_i .
- (d) The quantities in (a)–(c) using CD Modules 2.4 or 2.5.

Solution:

- (a) From Eq. (2.59),

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(100 - j100) - 50}{(100 - j100) + 50} = 0.62 \exp -j29.7^\circ.$$

(b) All formulae for Z_{in} require knowledge of $\beta = \omega/u_p$. Since the line is an air line, $u_p = c$, and from the expression for $v_g(t)$ we conclude $\omega = 2\pi \times 10^9 \text{ rad/s}$. Therefore

$$\beta = \frac{2\pi \times 10^9 \text{ rad/s}}{3 \times 10^8 \text{ m/s}} = \frac{20\pi}{3} \text{ rad/m}.$$

Then, using Eq. (2.79),

$$\begin{aligned} Z_{\text{in}} &= Z_0 \left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) \\ &= 50 \left[\frac{(100 - j100) + j50 \tan \left(\frac{20\pi}{3} \text{ rad/m} \times 5 \text{ cm} \right)}{50 + j(100 - j100) \tan \left(\frac{20\pi}{3} \text{ rad/m} \times 5 \text{ cm} \right)} \right] \\ &= 50 \left[\frac{(100 - j100) + j50 \tan \left(\frac{\pi}{3} \text{ rad} \right)}{50 + j(100 - j100) \tan \left(\frac{\pi}{3} \text{ rad} \right)} \right] = (12.5 - j12.7) \Omega. \end{aligned}$$

- (c) In phasor domain, $\tilde{V}_g = 5 \text{ V} \exp j0^\circ$. From Eq. (2.80),

$$\tilde{V}_i = \frac{\tilde{V}_g Z_{\text{in}}}{Z_g + Z_{\text{in}}} = \frac{5 \times (12.5 - j12.7)}{50 + (12.5 - j12.7)} = 1.40 \exp -j34.0^\circ \quad (\text{V}),$$

and also from Eq. (2.80),

$$\tilde{I}_i = \frac{\tilde{V}_i}{Z_{\text{in}}} = \frac{1.4 \exp -j34.0^\circ}{(12.5 - j12.7)} = 78.4 \exp j11.5^\circ \quad (\text{mA}).$$

Module 2.4
Transmission Line Simulator

Options:
Set Input / Output

d =

$d = 0.1666 \lambda = 49.98 \text{ mm}$
 $Z_L = 100.0 - j 100.0 \ \Omega$

$Z_g = 50.0 + j 0.0 \ \Omega$
 $Z_0 = 50.0 + j 0.0 \ \Omega$
 $f = 1.0 \text{ GHz}$

$V_g = 5.0 + j 0.0 \text{ V}$
 $\epsilon_r = 1.0$
 $\lambda = 300.0 \text{ mm}$

$d = 0.166667 \lambda = 50.0 \text{ mm}$
 $d = 0$

Set Line
Length units:
[λ]
[m]

Low Loss Approximation

Characteristic Impedance
 $Z_0 = 50 \ \Omega$

Frequency
 $f = 1\text{E}9 \text{ Hz}$

Relative Permittivity
 $\epsilon_r = 1.0$

Line Length
 $l = .05 \text{ [m]}$

Update

$Z_L = 100 + j -100 \ \Omega$

☒ Impedance
☐ Admittance

Update

Set Generator

$V_g = 5 + j 0.0 \text{ V}$

$Z_g = 50 + j 0.0 \ \Omega$

Update

Output
Transmission Line Data 1

Cursor
 $d = 0.1666 \lambda = 49.98 \text{ mm}$

Impedance
 $Z(d) = 12.530782 - j 12.743838 \ \Omega$

Admittance
 $Y(d) = 0.039229 + j 0.039896 \text{ [S]}$

Reflection Coefficient
 $\Gamma_d = -0.53543815 - j 0.31292389$

Voltage
 $V(d) = 1.161077 - j 0.782796 \text{ [V]}$

Current
 $I(d) = 0.076778 + j 0.015614 \text{ [A]}$

Power Flow
 $P_{av} = 38.461538 \text{ [m W]}$