2.38 The input impedance of a 31-cm–long lossless transmission line of unknown characteristic impedance was measured at 1 MHz. With the line terminated in a short circuit, the measurement yielded an input impedance equivalent to an inductor with inductance of 0.064 μH, and when the line was open-circuited, the measurement yielded an input impedance equivalent to a capacitor with capacitance of 40 pF. Find Z_0 of the line, the phase velocity, and the relative permittivity of the insulating material.

Solution: Now $\omega = 2\pi f = 6.28 \times 10^6$ rad/s, so

$$Z_{\text{in}}^{\text{sc}} = j\omega L = j2\pi \times 10^6 \times 0.064 \times 10^{-6} = j0.4\ \Omega$$

and $Z_{\text{in}}^{\text{oc}} = 1/j\omega C = 1/(j2\pi \times 10^6 \times 40 \times 10^{-12}) = -j4000\ \Omega$.

From Eq. (2.94), $Z_0 = \sqrt{Z_{\text{in}}^{\text{sc}} Z_{\text{in}}^{\text{oc}}} = \sqrt{(j0.4\ \Omega)(-j4000\ \Omega)} = 40\ \Omega$. Using Eq. (2.48),

$$\begin{aligned} u_{\text{p}} = \frac{\omega}{\beta} &= \frac{\omega l}{\tan^{-1}\sqrt{-Z_{\text{in}}^{\text{sc}}/Z_{\text{in}}^{\text{oc}}}} \\ &= \frac{6.28 \times 10^6 \times 0.31}{\tan^{-1}\left(\pm\sqrt{-j0.4/(-j4000)}\right)} = \frac{1.95 \times 10^6}{(\pm 0.01 + n\pi)} \text{ m/s}, \end{aligned}$$

where $n \geq 0$ for the plus sign and $n \geq 1$ for the minus sign. For $n = 0$, $u_{\text{p}} = 1.94 \times 10^8$ m/s $= 0.65c$ and $\varepsilon_{\text{r}} = (c/u_{\text{p}})^2 = 1/0.65^2 = 2.4$. For other values of n, u_{p} is very slow and ε_{r} is unreasonably high.