

2.42 A generator with $\tilde{V}_g = 300$ V and $Z_g = 50 \Omega$ is connected to a load $Z_L = 75 \Omega$ through a $50\text{-}\Omega$ lossless line of length $l = 0.15\lambda$.

- Compute Z_{in} , the input impedance of the line at the generator end.
- Compute \tilde{I}_i and \tilde{V}_i .
- Compute the time-average power delivered to the line, $P_{\text{in}} = \frac{1}{2} \Re[\tilde{V}_i \tilde{I}_i^*]$.
- Compute \tilde{V}_L , \tilde{I}_L , and the time-average power delivered to the load, $P_L = \frac{1}{2} \Re[\tilde{V}_L \tilde{I}_L^*]$. How does P_{in} compare to P_L ? Explain.
- Compute the time-average power delivered by the generator, P_g , and the time-average power dissipated in Z_g . Is conservation of power satisfied?

Solution:

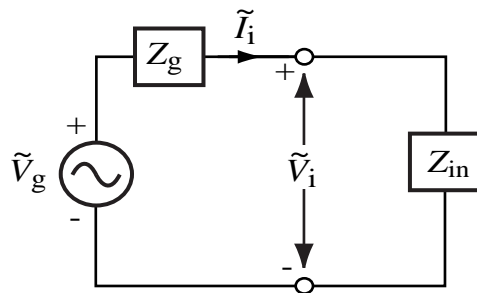
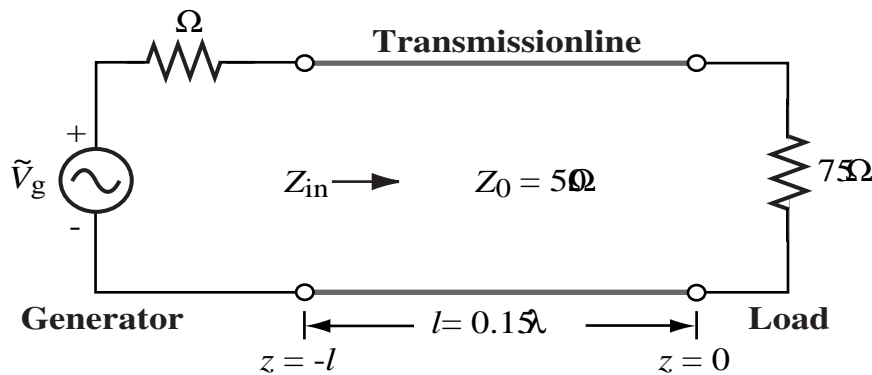


Figure P2.42 Circuit for Problem 2.42.

(a)

$$\beta l = \frac{2\pi}{\lambda} \times 0.15\lambda = 54^\circ,$$

$$Z_{\text{in}} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right] = 50 \left[\frac{75 + j50 \tan 54^\circ}{50 + j75 \tan 54^\circ} \right] = (41.25 - j16.35) \Omega.$$

(b)

$$\tilde{I}_i = \frac{\tilde{V}_g}{Z_g + Z_{\text{in}}} = \frac{300}{50 + (41.25 - j16.35)} = 3.24 e^{j10.16^\circ} \quad (\text{A}),$$

$$\tilde{V}_i = \tilde{I}_i Z_{\text{in}} = 3.24 e^{j10.16^\circ} (41.25 - j16.35) = 143.6 e^{-j11.46^\circ} \quad (\text{V}).$$

(c)

$$\begin{aligned} P_{\text{in}} &= \frac{1}{2} \Re[\tilde{V}_i \tilde{I}_i^*] = \frac{1}{2} \Re[143.6 e^{-j11.46^\circ} \times 3.24 e^{-j10.16^\circ}] \\ &= \frac{143.6 \times 3.24}{2} \cos(21.62^\circ) = 216 \quad (\text{W}). \end{aligned}$$

(d)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{75 - 50}{75 + 50} = 0.2,$$

$$V_0^+ = \tilde{V}_i \left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}} \right) = \frac{143.6 e^{-j11.46^\circ}}{e^{j54^\circ} + 0.2 e^{-j54^\circ}} = 150 e^{-j54^\circ} \quad (\text{V}),$$

$$\tilde{V}_L = V_0^+ (1 + \Gamma) = 150 e^{-j54^\circ} (1 + 0.2) = 180 e^{-j54^\circ} \quad (\text{V}),$$

$$\tilde{I}_L = \frac{V_0^+}{Z_0} (1 - \Gamma) = \frac{150 e^{-j54^\circ}}{50} (1 - 0.2) = 2.4 e^{-j54^\circ} \quad (\text{A}),$$

$$P_L = \frac{1}{2} \Re[\tilde{V}_L \tilde{I}_L^*] = \frac{1}{2} \Re[180 e^{-j54^\circ} \times 2.4 e^{j54^\circ}] = 216 \quad (\text{W}).$$

$P_L = P_{\text{in}}$, which is as expected because the line is lossless; power input to the line ends up in the load.

(e)

Power delivered by generator:

$$P_g = \frac{1}{2} \Re[\tilde{V}_g \tilde{I}_i] = \frac{1}{2} \Re[300 \times 3.24 e^{j10.16^\circ}] = 486 \cos(10.16^\circ) = 478.4 \quad (\text{W}).$$

Power dissipated in Z_g :

$$P_{Z_g} = \frac{1}{2} \Re[\tilde{I}_i \tilde{V}_{Z_g}] = \frac{1}{2} \Re[\tilde{I}_i \tilde{I}_i^* Z_g] = \frac{1}{2} |\tilde{I}_i|^2 Z_g = \frac{1}{2} (3.24)^2 \times 50 = 262.4 \quad (\text{W}).$$

Note 1: $P_g = P_{Z_g} + P_{\text{in}} = 478.4 \text{ W}$.
