2.45 The circuit shown in Fig. P2.45 consists of a 100-Ω lossless transmission line terminated in a load with $Z_L = (50 + j100)\ \Omega$. If the peak value of the load voltage was measured to be $|\widetilde{V}_L| = 12$ V, determine:

(a) the time-average power dissipated in the load,

(b) the time-average power incident on the line,

(c) the time-average power reflected by the load.

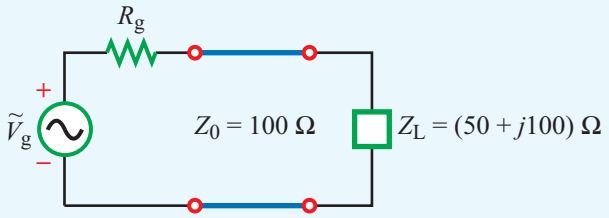

Figure P2.45 Circuit for Problem 2.45.

Solution:

(a)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 + j100 - 100}{50 + j100 + 100} = \frac{-50 + j100}{150 + j100} = 0.62e^{j82.9^\circ}.$$

The time average power dissipated in the load is:

$$\begin{aligned} P_{av} &= \frac{1}{2}|\tilde{I}_L|^2 R_L \\ &= \frac{1}{2}\left|\frac{\widetilde{V}_L}{Z_L}\right|^2 R_L \\ &= \frac{1}{2}\frac{|\widetilde{V}_L|^2}{|Z_L|^2}R_L = \frac{1}{2}\times 12^2 \times \frac{50}{50^2 + 100^2} = 0.29\ \text{W}. \end{aligned}$$

(b)

$$P_{av} = P^i_{av}(1 - |\Gamma|^2)$$

Hence,

$$P^i_{av} = \frac{P_{av}}{1 - |\Gamma|^2} = \frac{0.29}{1 - 0.62^2} = 0.47\ \text{W}.$$

(c)

$$P^r_{av} = -|\Gamma|^2 P^i_{av} = -(0.62)^2 \times 0.47 = -0.18\ \text{W}.$$