3.12 Two lines in the x–y plane are described by the expressions:

$$\begin{array}{ll} \text{Line 1} & x + 2y = -6, \\ \text{Line 2} & 3x + 4y = 8. \end{array}$$

Use vector algebra to find the smaller angle between the lines at their intersection point.

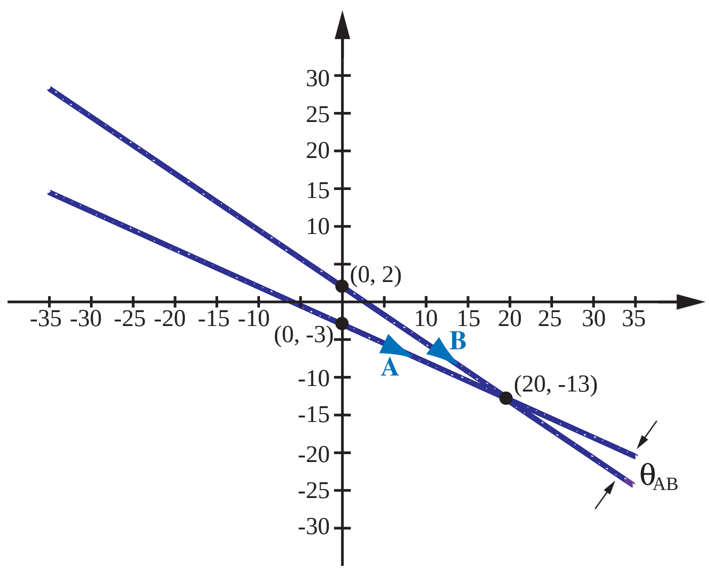

Figure P3.12 Lines 1 and 2.

Solution: Intersection point is found by solving the two equations simultaneously:

$$-2x - 4y = 12,$$
$$3x + 4y = 8.$$

The sum gives $x = 20$, which, when used in the first equation, gives $y = -13$.

Hence, intersection point is $(20, -13)$.

Another point on line 1 is $x = 0,\ y = -3$. Vector **A** from $(0, -3)$ to $(20, -13)$ is

$$\mathbf{A} = \hat{\mathbf{x}}(20) + \hat{\mathbf{y}}(-13 + 3) = \hat{\mathbf{x}}20 - \hat{\mathbf{y}}10,$$
$$|\mathbf{A}| = \sqrt{20^2 + 10^2} = \sqrt{500}.$$

A point on line 2 is $x = 0,\ y = 2$. Vector **B** from $(0, 2)$ to $(20, -13)$ is

$$\mathbf{B} = \hat{\mathbf{x}}(20) + \hat{\mathbf{y}}(-13 - 2) = \hat{\mathbf{x}}20 - \hat{\mathbf{y}}15,$$

$$|\mathbf{B}| = \sqrt{20^2 + 15^2} = \sqrt{625}.$$

Angle between **A** and **B** is

$$\theta_{AB} = \cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{|\mathbf{A}||\mathbf{B}|}\right) = \cos^{-1}\left(\frac{400+150}{\sqrt{500}\cdot\sqrt{625}}\right) = 10.3^\circ.$$