3.17 Find a vector $\mathbf{G}$ whose magnitude is 4 and whose direction is perpendicular to both vectors $\mathbf{E}$ and $\mathbf{F}$, where $\mathbf{E} = \hat{\mathbf{x}} + \hat{\mathbf{y}}2 - \hat{\mathbf{z}}2$ and $\mathbf{F} = \hat{\mathbf{y}}3 - \hat{\mathbf{z}}6$.

Solution: The cross product of two vectors produces a third vector which is perpendicular to both of the original vectors. Two vectors exist that satisfy the stated conditions, one along $\mathbf{E} \times \mathbf{F}$ and another along the opposite direction. Hence,

$$\begin{aligned}
\mathbf{G} = \pm 4 \frac{\mathbf{E} \times \mathbf{F}}{|\mathbf{E} \times \mathbf{F}|} &= \pm 4 \frac{(\hat{\mathbf{x}} + \hat{\mathbf{y}}2 - \hat{\mathbf{z}}2) \times (\hat{\mathbf{y}}3 - \hat{\mathbf{z}}6)}{|(\hat{\mathbf{x}} + \hat{\mathbf{y}}2 - \hat{\mathbf{z}}2) \times (\hat{\mathbf{y}}3 - \hat{\mathbf{z}}6)|} \\
&= \pm 4 \frac{(-\hat{\mathbf{x}}6 + \hat{\mathbf{y}}6 + \hat{\mathbf{z}}3)}{\sqrt{36 + 36 + 9}} \\
&= \pm \frac{4}{9} (-\hat{\mathbf{x}}6 + \hat{\mathbf{y}}6 + \hat{\mathbf{z}}3) = \pm \left(-\hat{\mathbf{x}}\frac{8}{3} + \hat{\mathbf{y}}\frac{8}{3} + \hat{\mathbf{z}}\frac{4}{3} \right).
\end{aligned}$$