3.20 When sketching or demonstrating the spatial variation of a vector field, we often use arrows, as in Fig. P3.20, wherein the length of the arrow is made to be proportional to the strength of the field and the direction of the arrow is the same as that of the field's. The sketch shown in Fig. P3.20, which represents the vector field $\mathbf{E} = \hat{\mathbf{r}}r$, consists of arrows pointing radially away from the origin and their lengths increase linearly in proportion to their distance away from the origin. Using this arrow representation, sketch each of the following vector fields:

(a) $\mathbf{E}_1 = -\hat{\mathbf{x}}y$,

(b) $\mathbf{E}_2 = \hat{\mathbf{y}}x$,

(c) $\mathbf{E}_3 = \hat{\mathbf{x}}x + \hat{\mathbf{y}}y$,

(d) $\mathbf{E}_4 = \hat{\mathbf{x}}x + \hat{\mathbf{y}}2y$,

(e) $\mathbf{E}_5 = \hat{\boldsymbol{\phi}}r$,

(f) $\mathbf{E}_6 = \hat{\mathbf{r}}\sin\phi$.

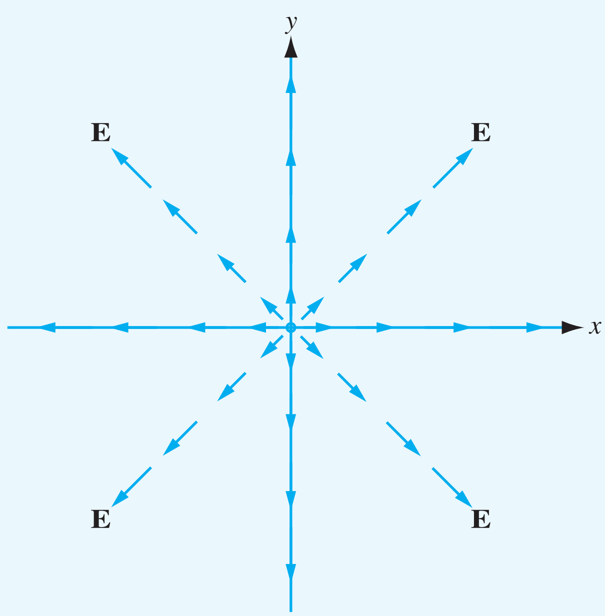
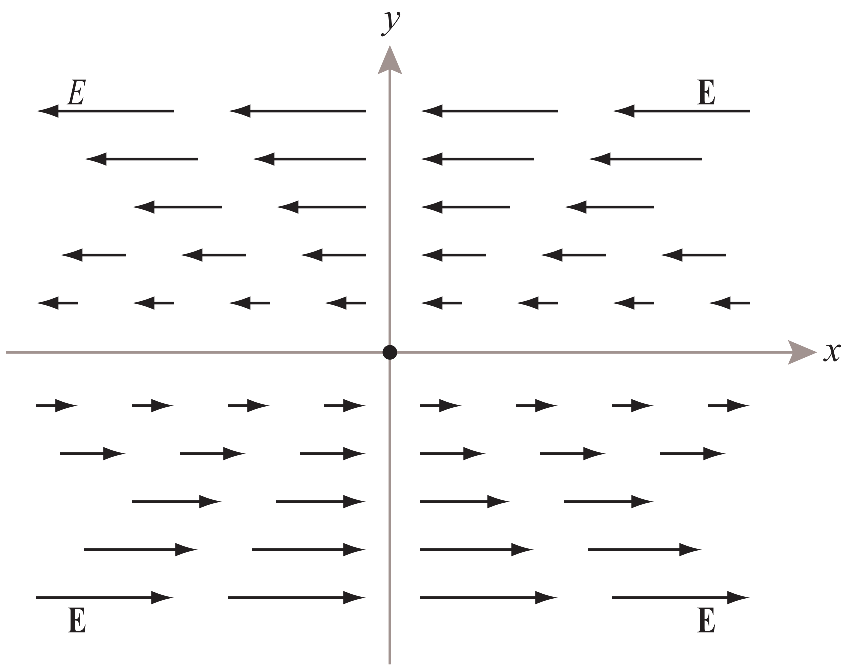


Figure P3.20 Arrow representation for vector field $\mathbf{E} = \hat{\mathbf{r}}r$ (Problem 3.20).

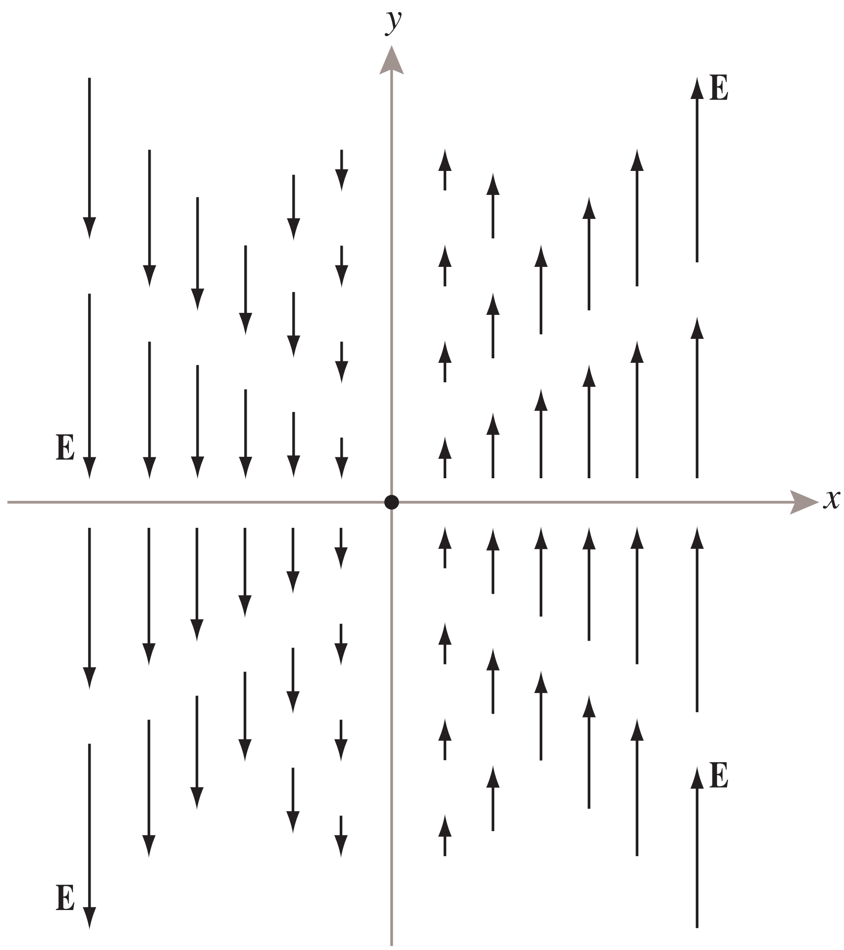
Solution:

(a)



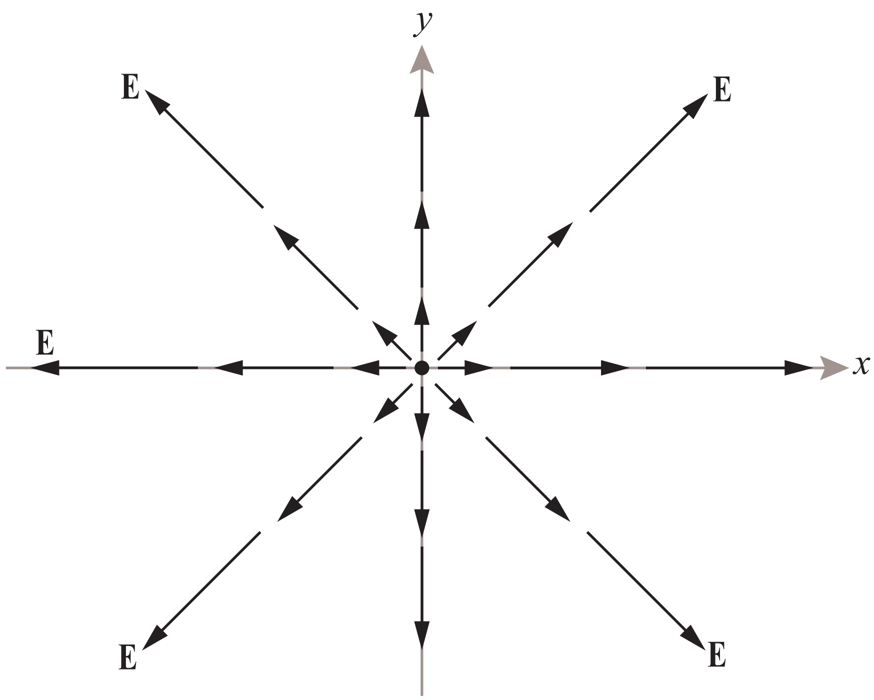
P3.20(a): $\mathbf{E}_1 = -\hat{\mathbf{x}}\, y$

(b)



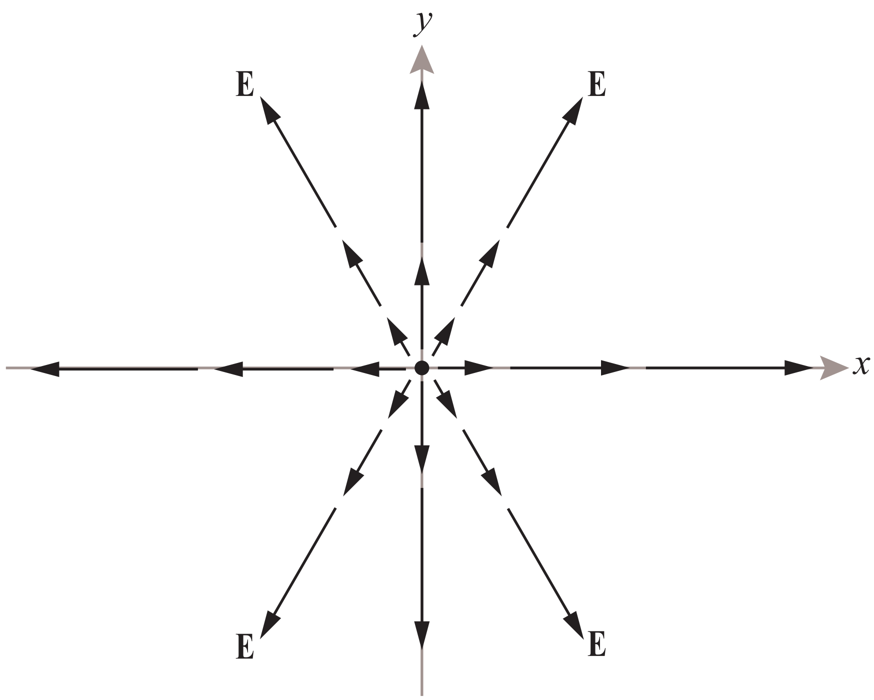
P3.20(b): $\mathbf{E}_2 = -\hat{\mathbf{y}}x$

(c)



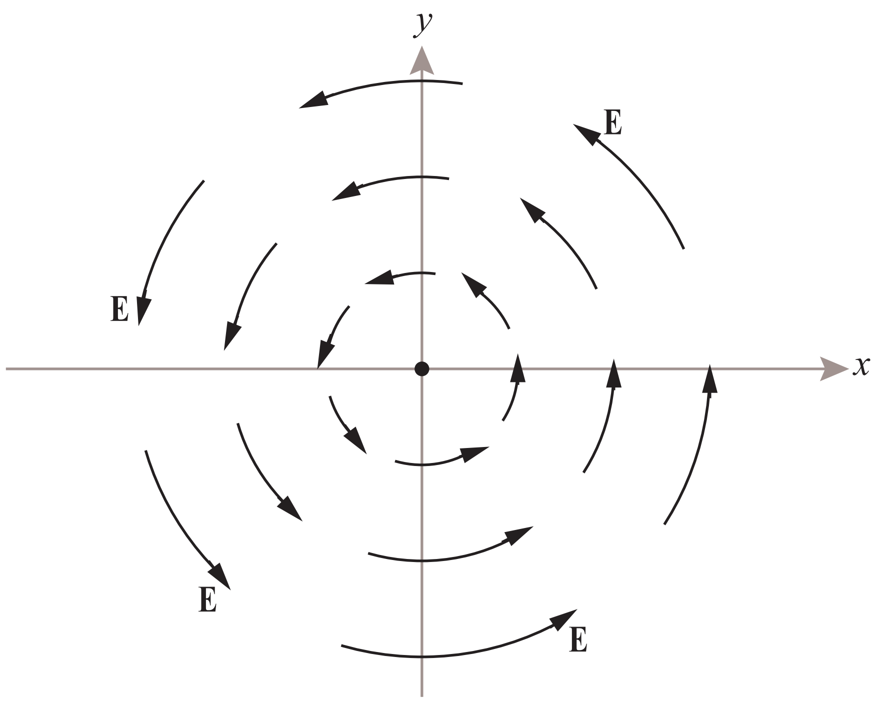
P3.20(c): $\mathbf{E}_3 = \hat{\mathbf{x}}x + \hat{\mathbf{y}}y$

(d)



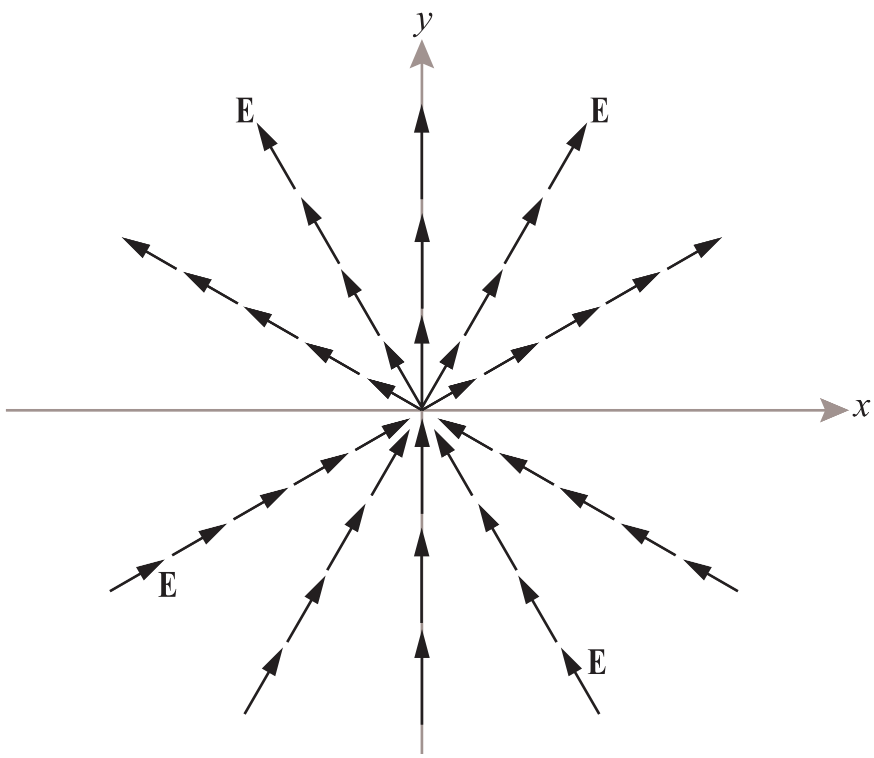
P3.20(d): $\mathbf{E}_4 = \hat{\mathbf{x}}x + \hat{\mathbf{y}}2y$

(e)



P3.20(e): $\mathbf{E}_5 = \hat{\boldsymbol{\phi}}\, r$

(f)



P3.20(f): $\mathbf{E}_6 = \hat{\mathbf{r}} \sin\phi$