3.22 Convert the coordinates of the following points from Cartesian to cylindrical and spherical coordinates:

(a) $P_1 = (1,2,0)$,
(b) $P_2 = (0,0,2)$,
(c) $P_3 = (1,1,3)$,
(d) $P_4 = (-2,2,-2)$.

Solution: Use the "coordinate variables" column in Table 3-2.

(a) In the cylindrical coordinate system,

$$P_1 = (\sqrt{1^2+2^2}, \tan^{-1}(2/1), 0) = (\sqrt{5}, 1.107 \text{ rad}, 0) \approx (2.24, 63.4^\circ, 0).$$

In the spherical coordinate system,

$$\begin{aligned} P_1 &= (\sqrt{1^2+2^2+0^2}, \tan^{-1}(\sqrt{1^2+2^2}/0), \tan^{-1}(2/1)) \\ &= (\sqrt{5}, \pi/2 \text{ rad}, 1.107 \text{ rad}) \approx (2.24, 90.0^\circ, 63.4^\circ). \end{aligned}$$

Note that in both the cylindrical and spherical coordinates, ϕ is in Quadrant I.

(b) In the cylindrical coordinate system,

$$P_2 = (\sqrt{0^2+0^2}, \tan^{-1}(0/0), 2) = (0, 0 \text{ rad}, 2) = (0, 0^\circ, 2).$$

In the spherical coordinate system,

$$\begin{aligned} P_2 &= \left(\sqrt{0^2+0^2+2^2}, \tan^{-1}\left(\sqrt{0^2+0^2}/2\right), \tan^{-1}(0/0)\right) \\ &= (2, 0 \text{ rad}, 0 \text{ rad}) = (2, 0^\circ, 0^\circ). \end{aligned}$$

Note that in both the cylindrical and spherical coordinates, ϕ is arbitrary and may take any value.

(c) In the cylindrical coordinate system,

$$P_3 = \left(\sqrt{1^2+1^2}, \tan^{-1}(1/1), 3\right) = \left(\sqrt{2}, \pi/4 \text{ rad}, 3\right) \approx (1.41, 45.0^\circ, 3).$$

In the spherical coordinate system,

$$\begin{aligned} P_3 &= \left(\sqrt{1^2+1^2+3^2}, \tan^{-1}\left(\sqrt{1^2+1^2}/3\right), \tan^{-1}(1/1)\right) \\ &= \left(\sqrt{11}, 0.44 \text{ rad}, \pi/4 \text{ rad}\right) \approx (3.32, 25.2^\circ, 45.0^\circ). \end{aligned}$$

Note that in both the cylindrical and spherical coordinates, ϕ is in Quadrant I.

(d) In the cylindrical coordinate system,

$$P_4 = \left(\sqrt{(-2)^2 + 2^2}, \tan^{-1}(2/-2), -2\right)$$
$$= \left(2\sqrt{2}, 3\pi/4 \text{ rad}, -2\right) \approx (2.83, 135.0^\circ, -2).$$

In the spherical coordinate system,

$$P_4 = \left(\sqrt{(-2)^2 + 2^2 + (-2)^2}, \tan^{-1}\left(\sqrt{(-2)^2 + 2^2}/-2\right), \tan^{-1}(2/-2)\right)$$
$$= \left(2\sqrt{3}, 2.187 \text{ rad}, 3\pi/4 \text{ rad}\right) \approx (3.46, 125.3^\circ, 135.0^\circ).$$

Note that in both the cylindrical and spherical coordinates, ϕ is in Quadrant II.