3.28 A vector field is given in cylindrical coordinates by

$$\mathbf{E} = \hat{\mathbf{r}} r \cos\phi + \hat{\boldsymbol{\phi}} r \sin\phi + \hat{\mathbf{z}} z^2.$$

Point $P = (2, \pi, 3)$ is located on the surface of the cylinder described by $r = 2$. At point P, find:

(a) the vector component of **E** perpendicular to the cylinder,

(b) the vector component of **E** tangential to the cylinder.

Solution:

(a) $\mathbf{E}_{\text{n}} = \hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \mathbf{E}) = \hat{\mathbf{r}}[\hat{\mathbf{r}} \cdot (\hat{\mathbf{r}} r \cos\phi + \hat{\boldsymbol{\phi}} r \sin\phi + \hat{\mathbf{z}} z^2)] = \hat{\mathbf{r}} r \cos\phi$.

At $P = (2, \pi, 3)$, $\mathbf{E}_{\text{n}} = \hat{\mathbf{r}} 2 \cos\pi = -\hat{\mathbf{r}} 2$.

(b) $\mathbf{E}_{\text{t}} = \mathbf{E} - \mathbf{E}_{\text{n}} = \hat{\boldsymbol{\phi}} r \sin\phi + \hat{\mathbf{z}} z^2$.

At $P = (2, \pi, 3)$, $\mathbf{E}_{\text{t}} = \hat{\boldsymbol{\phi}} 2 \sin\pi + \hat{\mathbf{z}} 3^2 = \hat{\mathbf{z}} 9$.