

3.3 In Cartesian coordinates, the three corners of a triangle are $P_1 = (0, 4, 4)$, $P_2 = (4, -4, 4)$, and $P_3 = (2, 2, -4)$. Find the area of the triangle.

Solution: Let $\vec{B} = \overrightarrow{P_1P_2} = \hat{x}4 - \hat{y}8$ and $\vec{C} = \overrightarrow{P_1P_3} = \hat{x}2 - \hat{y}2 - \hat{z}8$ represent two sides of the triangle. Since the magnitude of the cross product is the area of the parallelogram (see the definition of cross product in Section 3-1.4), half of this is the area of the triangle:

$$\begin{aligned} A &= \frac{1}{2} |\vec{B} \times \vec{C}| = \frac{1}{2} |(\hat{x}4 - \hat{y}8) \times (\hat{x}2 - \hat{y}2 - \hat{z}8)| \\ &= \frac{1}{2} |\hat{x}(-8)(-8) + \hat{y}(-4)(-8) + \hat{z}(4(-2) - (-8)2)| \\ &= \frac{1}{2} |\hat{x}64 + \hat{y}32 + \hat{z}8| = \frac{1}{2} \sqrt{64^2 + 32^2 + 8^2} = \frac{1}{2} \sqrt{5184} = 36, \end{aligned}$$

where the cross product is evaluated with Eq. (3.27).
