

3.31 Find the distance between the following pairs of points:

- (a) $P_1 = (1, 2, 3)$ and $P_2 = (-2, -3, -2)$ in Cartesian coordinates,
- (b) $P_3 = (1, \pi/4, 3)$ and $P_4 = (3, \pi/4, 4)$ in cylindrical coordinates,
- (c) $P_5 = (4, \pi/2, 0)$ and $P_6 = (3, \pi, 0)$ in spherical coordinates.

Solution:

(a)

$$d = [(-2 - 1)^2 + (-3 - 2)^2 + (-2 - 3)^2]^{1/2} = [9 + 25 + 25]^{1/2} = \sqrt{59} = 7.68.$$

(b)

$$\begin{aligned} d &= [r_2^2 + r_1^2 - 2r_1r_2 \cos(\phi_2 - \phi_1) + (z_2 - z_1)^2]^{1/2} \\ &= \left[9 + 1 - 2 \times 3 \times 1 \times \cos\left(\frac{\pi}{4} - \frac{\pi}{4}\right) + (4 - 3)^2 \right]^{1/2} \\ &= (10 - 6 + 1)^{1/2} = 5^{1/2} = 2.24. \end{aligned}$$

(c)

$$\begin{aligned} d &= \{R_2^2 + R_1^2 - 2R_1R_2[\cos \theta_2 \cos \theta_1 + \sin \theta_1 \sin \theta_2 \cos(\phi_2 - \phi_1)]\}^{1/2} \\ &= \left\{ 9 + 16 - 2 \times 3 \times 4 \left[\cos \pi \cos \frac{\pi}{2} + \sin \frac{\pi}{2} \sin \pi \cos(0 - 0) \right] \right\}^{1/2} \\ &= \{9 + 16 - 0\}^{1/2} = \sqrt{25} = 5. \end{aligned}$$
