3.37 For each of the following scalar fields, obtain an analytical solution for ∇T and generate a corresponding arrow representation.

(a) $T = 10 + x$, for $-10 \leq x \leq 10$

(b) $T = x^2$, for $-10 \leq x \leq 10$

(c) $T = 100 + xy$, for $-10 \leq x \leq 10$

(d) $T = x^2y^2$, for $-10 \leq x, y \leq 10$

(e) $T = 20 + x + y$, for $-10 \leq x, y \leq 10$

(f) $T = 1 + \sin(\pi x/3)$, for $-10 \leq x \leq 10$

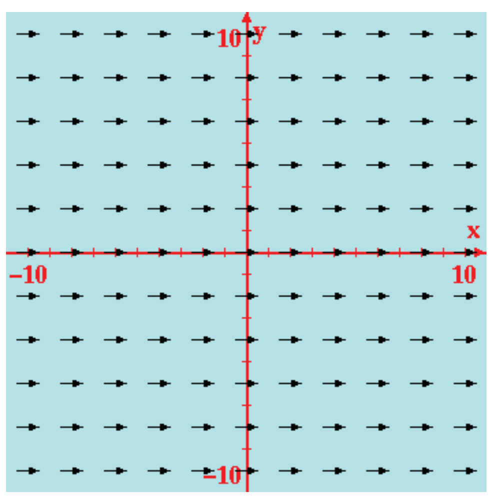
(g) $T = 1 + \cos(\pi x/3)$, for $-10 \leq x \leq 10$

(h) $T = 15 + r\cos\phi$, for $\begin{cases} 0 \leq r \leq 10 \\ 0 \leq \phi \leq 2\pi. \end{cases}$

(i) $T = 15 + r\cos^2\phi$, for $\begin{cases} 0 \leq r \leq 10 \\ 0 \leq \phi \leq 2\pi. \end{cases}$

Solution:

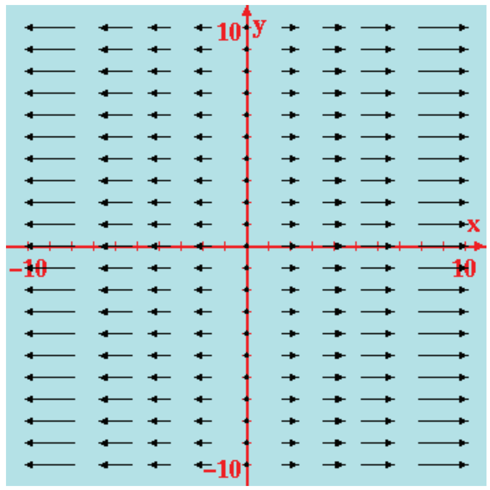
(a)



$$
\begin{aligned}
T &= 10 + x \\
\nabla T &= \hat{\mathbf{x}}\,\frac{\partial T}{\partial x} + \hat{\mathbf{y}}\,\frac{\partial T}{\partial y} + \hat{\mathbf{z}}\,\frac{\partial T}{\partial z} \\
&= \hat{\mathbf{x}}.
\end{aligned}
$$

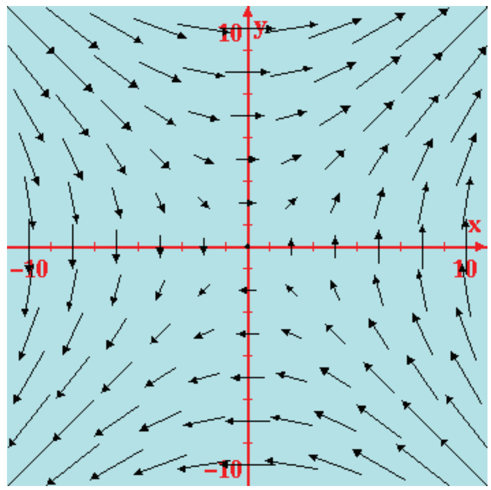
The direction of ∇T displays the fact that T increases linearly with x only.

(b)



$$
\begin{aligned}
T &= x^2 \\
\nabla T &= \hat{\mathbf{x}}\,\frac{\partial T}{\partial x} + \hat{\mathbf{y}}\,\frac{\partial T}{\partial y} + \hat{\mathbf{z}}\,\frac{\partial T}{\partial z} \\
&= \hat{\mathbf{x}}\,2x.
\end{aligned}
$$

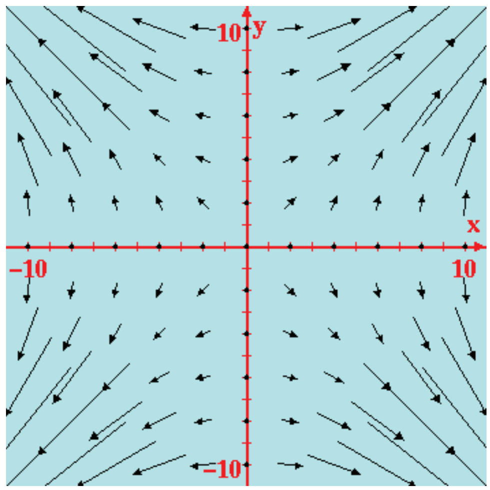
(c)



$$
\begin{aligned}
T &= 100 + xy \\
\nabla T &= \hat{\mathbf{x}}\,\frac{\partial T}{\partial x} + \hat{\mathbf{y}}\,\frac{\partial T}{\partial y} + \hat{\mathbf{z}}\,\frac{\partial T}{\partial z} \\
&= \hat{\mathbf{x}}\,y + \hat{\mathbf{y}}\,x.
\end{aligned}
$$

The magnitude of the gradient increases monotonically with x.

(d)

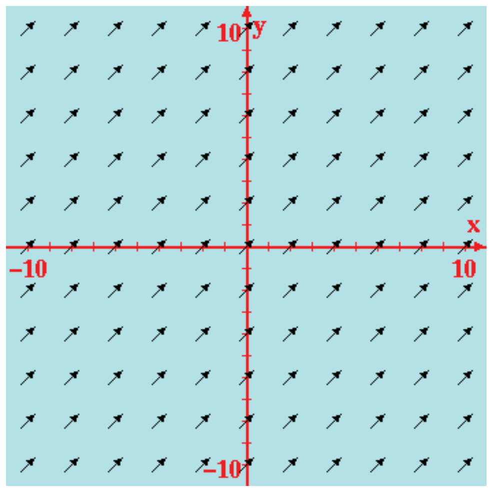


$$T = x^2y^2$$

$$\nabla T = \hat{\mathbf{x}}\,\frac{\partial T}{\partial x} + \hat{\mathbf{y}}\,\frac{\partial T}{\partial y} + \hat{\mathbf{z}}\,\frac{\partial T}{\partial z}$$

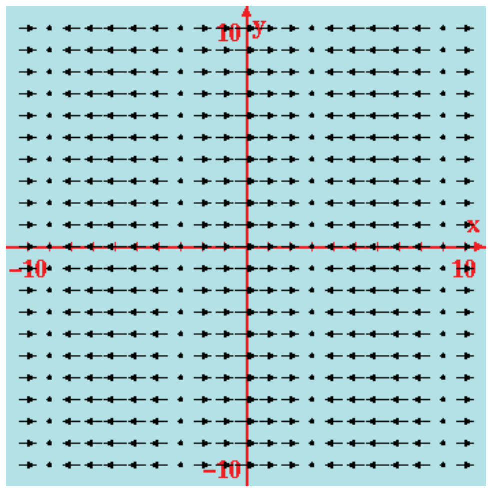
$$= \hat{\mathbf{x}}\,2xy^2 + \hat{\mathbf{y}}\,2x^2y.$$

(e)



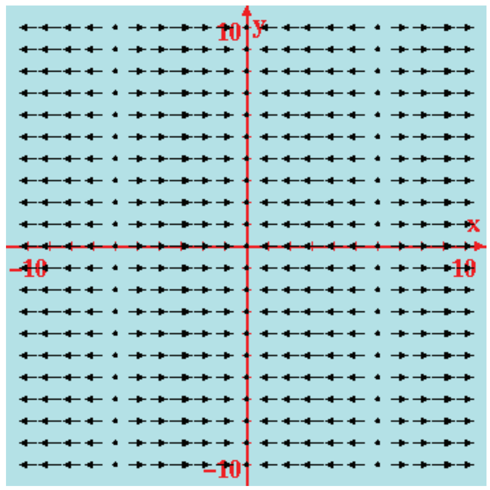
$$
\begin{aligned}
T &= 20 + x + y \\
\nabla T &= \hat{\mathbf{x}}\,\frac{\partial T}{\partial x} + \hat{\mathbf{y}}\,\frac{\partial T}{\partial y} + \hat{\mathbf{z}}\,\frac{\partial T}{\partial z} \\
&= \hat{\mathbf{x}} + \hat{\mathbf{y}}.
\end{aligned}
$$

(f)



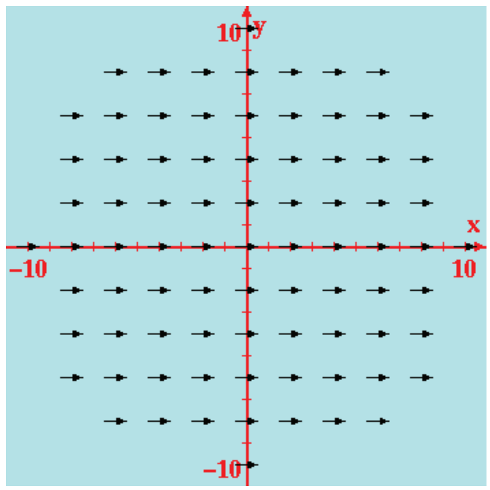
$$
\begin{aligned}
T &= 1 + \sin(\pi x/3) \\
\nabla T &= \hat{\mathbf{x}}\,\frac{\partial T}{\partial x} + \hat{\mathbf{y}}\,\frac{\partial T}{\partial y} + \hat{\mathbf{z}}\,\frac{\partial T}{\partial z} \\
&= \hat{\mathbf{x}}\,\frac{2\pi}{6}\cos(\pi x/3).
\end{aligned}
$$

(g)



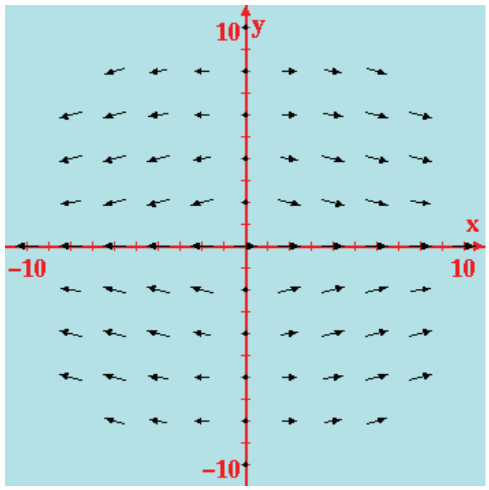
$$\begin{aligned} T &= 1 + \cos(\pi x/3) \\ \nabla T &= \hat{\mathbf{x}}\,\frac{\partial T}{\partial x} + \hat{\mathbf{y}}\,\frac{\partial T}{\partial y} + \hat{\mathbf{z}}\,\frac{\partial T}{\partial z} \\ &= -\hat{\mathbf{x}}\,\frac{2\pi}{6}\sin(\pi x/3). \end{aligned}$$

(h)



$$
\begin{aligned}
T &= 15 + r\cos\theta \\
\nabla T &= \hat{\mathbf{r}}\,\frac{\partial T}{\partial r} + \hat{\boldsymbol{\phi}}\,\frac{1}{r}\,\frac{\partial T}{\partial \phi} \\
&= \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi \\
&= \hat{\mathbf{x}}.
\end{aligned}
$$

(i)



$$
\begin{aligned}
T &= 15 + r\cos^2\theta \\
\nabla T &= \hat{\mathbf{r}}\,\frac{\partial T}{\partial r} + \hat{\boldsymbol{\phi}}\,\frac{1}{r}\,\frac{\partial T}{\partial \phi} \\
&= \hat{\mathbf{r}}\cos^2\phi - \hat{\boldsymbol{\phi}}\,2\sin\phi\cos\phi.
\end{aligned}
$$