3.39 Follow a procedure similar to that leading to Eq. (3.82) to derive the expression given by Eq. (3.83) for ∇ in spherical coordinates.

Solution: From the chain rule and Table 3-2,

$$
\begin{aligned}
\nabla T &= \hat{\mathbf{x}}\frac{\partial T}{\partial x} + \hat{\mathbf{y}}\frac{\partial T}{\partial y} + \hat{\mathbf{z}}\frac{\partial T}{\partial z} \\
&= \hat{\mathbf{x}}\left(\frac{\partial T}{\partial R}\frac{\partial R}{\partial x} + \frac{\partial T}{\partial \theta}\frac{\partial \theta}{\partial x} + \frac{\partial T}{\partial \phi}\frac{\partial \phi}{\partial x}\right) \\
&\quad + \hat{\mathbf{y}}\left(\frac{\partial T}{\partial R}\frac{\partial R}{\partial y} + \frac{\partial T}{\partial \theta}\frac{\partial \theta}{\partial y} + \frac{\partial T}{\partial \phi}\frac{\partial \phi}{\partial y}\right) \\
&\quad + \hat{\mathbf{z}}\left(\frac{\partial T}{\partial R}\frac{\partial R}{\partial z} + \frac{\partial T}{\partial \theta}\frac{\partial \theta}{\partial z} + \frac{\partial T}{\partial \phi}\frac{\partial \phi}{\partial z}\right) \\
&= \hat{\mathbf{x}}\left(\frac{\partial T}{\partial R}\frac{\partial}{\partial x}\sqrt{x^2+y^2+z^2} + \frac{\partial T}{\partial \theta}\frac{\partial}{\partial x}\tan^{-1}\left(\sqrt{x^2+y^2}/z\right) + \frac{\partial T}{\partial \phi}\frac{\partial}{\partial x}\tan^{-1}(y/x)\right) \\
&+ \hat{\mathbf{y}}\left(\frac{\partial T}{\partial R}\frac{\partial}{\partial y}\sqrt{x^2+y^2+z^2} + \frac{\partial T}{\partial \theta}\frac{\partial}{\partial y}\tan^{-1}\left(\sqrt{x^2+y^2}/z\right) + \frac{\partial T}{\partial \phi}\frac{\partial}{\partial y}\tan^{-1}(y/x)\right) \\
&+ \hat{\mathbf{z}}\left(\frac{\partial T}{\partial R}\frac{\partial}{\partial z}\sqrt{x^2+y^2+z^2} + \frac{\partial T}{\partial \theta}\frac{\partial}{\partial z}\tan^{-1}\left(\sqrt{x^2+y^2}/z\right) + \frac{\partial T}{\partial \phi}\frac{\partial}{\partial z}\tan^{-1}(y/x)\right) \\
&= \hat{\mathbf{x}}\left(\frac{\partial T}{\partial R}\frac{x}{\sqrt{x^2+y^2+z^2}} + \frac{\partial T}{\partial \theta}\frac{z}{x^2+y^2+z^2}\frac{x}{\sqrt{x^2+y^2}} + \frac{\partial T}{\partial \phi}\frac{-y}{x^2+y^2}\right) \\
&\quad + \hat{\mathbf{y}}\left(\frac{\partial T}{\partial R}\frac{y}{\sqrt{x^2+y^2+z^2}} + \frac{\partial T}{\partial \theta}\frac{z}{x^2+y^2+z^2}\frac{y}{\sqrt{x^2+y^2}} + \frac{\partial T}{\partial \phi}\frac{x}{x^2+y^2}\right) \\
&\quad + \hat{\mathbf{z}}\left(\frac{\partial T}{\partial R}\frac{z}{\sqrt{x^2+y^2+z^2}} + \frac{\partial T}{\partial \theta}\frac{-1}{x^2+y^2+z^2}\sqrt{x^2+y^2} + \frac{\partial T}{\partial \phi}0\right) \\
&= \hat{\mathbf{x}}\left(\frac{\partial T}{\partial R}\frac{R\sin\theta\cos\phi}{R} + \frac{\partial T}{\partial \theta}\frac{R\cos\theta}{R^2}\frac{R\sin\theta\cos\phi}{R\sin\theta} + \frac{\partial T}{\partial \phi}\frac{-R\sin\theta\sin\phi}{R^2\sin^2\theta}\right) \\
&\quad + \hat{\mathbf{y}}\left(\frac{\partial T}{\partial R}\frac{R\sin\theta\sin\phi}{R} + \frac{\partial T}{\partial \theta}\frac{R\cos\theta}{R^2}\frac{R\sin\theta\sin\phi}{R\sin\theta} + \frac{\partial T}{\partial \phi}\frac{R\sin\theta\cos\phi}{R^2\sin^2\theta}\right) \\
&\quad + \hat{\mathbf{z}}\left(\frac{\partial T}{\partial R}\frac{R\cos\theta}{R} + \frac{\partial T}{\partial \theta}\frac{-R\sin\theta}{R^2}\right) \\
&= \hat{\mathbf{x}}\left(\frac{\partial T}{\partial R}\sin\theta\cos\phi + \frac{\partial T}{\partial \theta}\frac{\cos\theta\cos\phi}{R} + \frac{\partial T}{\partial \phi}\frac{-\sin\phi}{R\sin\theta}\right)
\end{aligned}
$$

$$
\begin{aligned}
&+\hat{\mathbf{y}}\left(\frac{\partial T}{\partial R}\sin\theta\sin\phi+\frac{\partial T}{\partial\theta}\frac{\cos\theta\sin\phi}{R}+\frac{\partial T}{\partial\phi}\frac{\cos\phi}{R\sin\theta}\right)\\
&+\hat{\mathbf{z}}\left(\frac{\partial T}{\partial R}\cos\theta+\frac{\partial T}{\partial\theta}\frac{-\sin\theta}{R}\right)\\
=&\,(\hat{\mathbf{x}}\sin\theta\cos\phi+\hat{\mathbf{y}}\sin\theta\sin\phi+\hat{\mathbf{z}}\cos\theta)\,\frac{\partial T}{\partial R}\\
&+(\hat{\mathbf{x}}\cos\theta\cos\phi+\hat{\mathbf{y}}\cos\theta\sin\phi-\hat{\mathbf{z}}\sin\theta)\,\frac{1}{R}\frac{\partial T}{\partial\theta}\\
&+(-\hat{\mathbf{x}}\sin\phi+\hat{\mathbf{y}}\cos\phi)\,\frac{1}{R\sin\theta}\frac{\partial T}{\partial\phi}\\
=&\,\hat{\mathbf{R}}\frac{\partial T}{\partial R}+\hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial T}{\partial\theta}+\hat{\boldsymbol{\phi}}\frac{1}{R\sin\theta}\frac{\partial T}{\partial\phi}\,,
\end{aligned}
$$

which is Eq. (3.83).