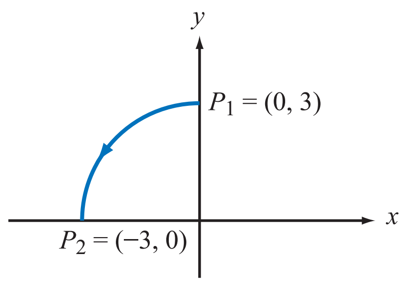
3.41 Evaluate the line integral of $\mathbf{E} = \hat{\mathbf{x}}x - \hat{\mathbf{y}}y$ along the segment P_1 to P_2 of the circular path shown in Fig. P3.41.



Solution: We need to calculate:

$$\int_{P_1}^{P_2} \mathbf{E} \cdot d\boldsymbol{\ell}.$$

Since the path is along the perimeter of a circle, it is best to use cylindrical coordinates, which requires expressing both $\mathbf{E}$ and $d\boldsymbol{\ell}$ in cylindrical coordinates. Using Table 3-2,

$$\begin{aligned}\mathbf{E} = \hat{\mathbf{x}}x - \hat{\mathbf{y}}y &= (\hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi)r\cos\phi - (\hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi)r\sin\phi \\ &= \hat{\mathbf{r}}\,r(\cos^2\phi - \sin^2\phi) - \hat{\boldsymbol{\phi}}2r\sin\phi\cos\phi\end{aligned}$$

The designated path is along the ϕ-direction at a constant $r = 3$. From Table 3-1, the applicable component of $d\boldsymbol{\ell}$ is:

$$d\boldsymbol{\ell} = \hat{\boldsymbol{\phi}}\,r\,d\phi.$$

Hence,

$$\begin{aligned}\int_{P_1}^{P_2} \mathbf{E} \cdot d\boldsymbol{\ell} &= \int_{\phi=90^\circ}^{\phi=180^\circ} \left[\hat{\mathbf{r}}\,r(\cos^2\phi - \sin^2\phi) - \hat{\boldsymbol{\phi}}\,2r\sin\phi\cos\phi\right] \cdot \hat{\boldsymbol{\phi}}\,r\,d\phi \Big|_{r=3} \\ &= \int_{90^\circ}^{180^\circ} -2r^2\sin\phi\cos\phi\;d\phi\Big|_{r=3} \\ &= -2r^2 \left.\left.\frac{\sin^2\phi}{2}\right|_{\phi=90^\circ}^{180^\circ}\right|_{r=3} = 9.\end{aligned}$$