3.42 For the scalar function $T = \frac{1}{2}e^{-r/5}\cos\phi$, determine its directional derivative along the radial direction $\hat{\mathbf{r}}$ and then evaluate it at $P = (2, \pi/4, 3)$.

Solution:

$$T = \frac{1}{2}e^{-r/5}\cos\phi,$$

$$\nabla T = \hat{\mathbf{r}}\frac{\partial T}{\partial r} + \hat{\boldsymbol{\phi}}\frac{1}{r}\frac{\partial T}{\partial \phi} + \hat{\mathbf{z}}\frac{\partial T}{\partial z} = -\hat{\mathbf{r}}\frac{e^{-r/5}\cos\phi}{10} - \hat{\boldsymbol{\phi}}\frac{e^{-r/5}\sin\phi}{2r},$$

$$\frac{dT}{dl} = \nabla T \cdot \hat{\mathbf{r}} = -\frac{e^{-r/5}\cos\phi}{10},$$

$$\left.\frac{dT}{dl}\right|_{(2,\pi/4,3)} = -\frac{e^{-2/5}\cos\frac{\pi}{4}}{10} = -4.74 \times 10^{-2}.$$