3.5 Given vectors $\mathbf{A} = \hat{\mathbf{x}} + \hat{\mathbf{y}}2 - \hat{\mathbf{z}}3$, $\mathbf{B} = \hat{\mathbf{x}}2 - \hat{\mathbf{y}}4$, and $\mathbf{C} = \hat{\mathbf{y}}2 - \hat{\mathbf{z}}4$, find

(a) A and $\hat{\mathbf{a}}$,

(b) the component of $\mathbf{B}$ along $\mathbf{C}$,

(c) θ_{AC},

(d) $\mathbf{A} \times \mathbf{C}$,

(e) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$,

(f) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$,

(g) $\hat{\mathbf{x}} \times \mathbf{B}$, and

(h) $(\mathbf{A} \times \hat{\mathbf{y}}) \cdot \hat{\mathbf{z}}$.

Solution:

(a) From Eq. (3.4),

$$A = \sqrt{1^2 + 2^2 + (-3)^2} = \sqrt{14},$$

and, from Eq. (3.5),

$$\hat{\mathbf{a}}_A = \frac{\hat{\mathbf{x}} + \hat{\mathbf{y}}2 - \hat{\mathbf{z}}3}{\sqrt{14}}.$$

(b) The component of $\vec{B}$ along $\vec{C}$ (see Section 3-1.4) is given by

$$B\cos\theta_{BC} = \frac{\vec{B}\cdot\vec{C}}{C} = \frac{-8}{\sqrt{20}} = -1.8\,.$$

(c) From Eq. (3.18),

$$\theta_{AC} = \cos^{-1}\frac{\vec{A}\cdot\vec{C}}{AC} = \cos^{-1}\frac{4+12}{\sqrt{14}\sqrt{20}} = \cos^{-1}\frac{16}{\sqrt{280}} = 17.0^\circ.$$

(d) From Eq. (3.27),

$$\vec{A}\times\vec{C} = \hat{\mathbf{x}}\,(2\,(-4) - (-3)\,2) + \hat{\mathbf{y}}\,((-3)\,0 - 1\,(-4)) + \hat{\mathbf{z}}\,(1\,(2) - 2\,(0)) = -\hat{\mathbf{x}}2 + \hat{\mathbf{y}}4 + \hat{\mathbf{z}}2.$$

(e) From Eq. (3.27) and Eq. (3.21),

$$\vec{A}\cdot\left(\vec{B}\times\vec{C}\right) = \vec{A}\cdot(\hat{\mathbf{x}}16 + \hat{\mathbf{y}}8 + \hat{\mathbf{z}}4) = 1\,(16) + 2\,(8) + (-3)\,4 = 20.$$

Eq. (3.30) could also have been used in the solution. Also, Eq. (3.29) could be used in conjunction with the result of part (d).

(f) By repeated application of Eq. (3.27),

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right) = \vec{A}\times(\hat{\mathbf{x}}16 + \hat{\mathbf{y}}8 + \hat{\mathbf{z}}4) = \hat{\mathbf{x}}32 - \hat{\mathbf{y}}52 - \hat{\mathbf{z}}24.$$

Eq. (3.33) could also have been used.

(g) From Eq. (3.27),

$$\hat{\mathbf{x}} \times \vec{B} = -\hat{\mathbf{z}}4.$$

(h) From Eq. (3.27) and Eq. (3.21),

$$\left(\vec{A} \times \hat{\mathbf{y}}\right) \cdot \hat{\mathbf{z}} = (\hat{\mathbf{x}}3 + \hat{\mathbf{z}}) \cdot \hat{\mathbf{z}} = 1.$$

Eq. (3.29) and Eq. (3.25) could also have been used in the solution.