4.14 A line of charge with uniform density $\rho_\ell = 8$ (μC/m) exists in air along the z-axis between $z = 0$ and $z = 5$ cm. Find $\mathbf{E}$ at (0,10 cm,0).

Solution: Use of Eq. (4.21c) for the line of charge shown in Fig. P4.14 gives

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int_{l'} \hat{\mathbf{R}}' \frac{\rho_l\, dl'}{R'^2},$$

$$R' = \hat{\mathbf{y}}0.1 - \hat{\mathbf{z}}z$$

$$= \frac{1}{4\pi\varepsilon_0}\int_{z=0}^{0.05} (8\times 10^{-6}) \frac{(\hat{\mathbf{y}}0.1 - \hat{\mathbf{z}}z)}{[(0.1)^2 + z^2]^{3/2}}\, dz$$

$$= \frac{8\times 10^{-6}}{4\pi\varepsilon_0}\left[\frac{\hat{\mathbf{y}}10z + \hat{\mathbf{z}}}{\sqrt{(0.1)^2 + z^2}}\right]\Bigg|_{z=0}^{0.05}$$

$$= 71.86\times 10^3\,[\hat{\mathbf{y}}4.47 - \hat{\mathbf{z}}1.06] = \hat{\mathbf{y}}321.4\times 10^3 - \hat{\mathbf{z}}76.2\times 10^3 \quad \text{(V/m)}.$$

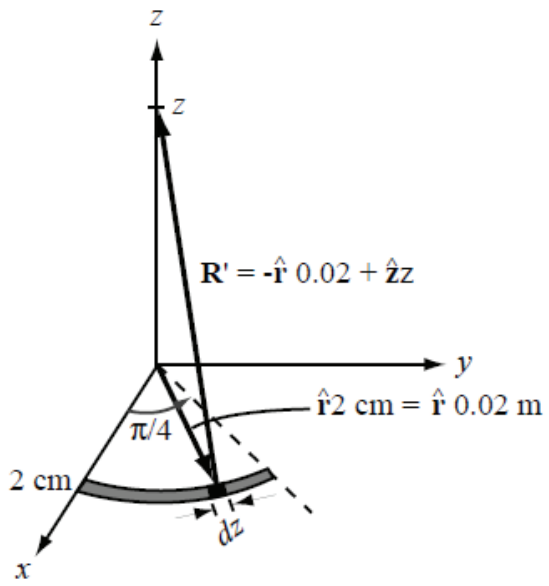

Figure P4.14 Line charge.