4.15 Electric charge is distributed along an arc located in the x–y plane and defined by $r = 2$ cm and $0 \leq \phi \leq \pi/4$. If $\rho_\ell = 5$ (μC/m), find $\mathbf{E}$ at $(0,0,z)$ and then evaluate it at:

(a) The origin.

(b) $z = 5$ cm

(c) $z = -5$ cm

Solution: For the arc of charge shown in Fig. P4.15, $dl = r\,d\phi = 0.02\,d\phi$, and $\mathbf{R}' = -\hat{\mathbf{x}}0.02\cos\phi - \hat{\mathbf{y}}0.02\sin\phi + \hat{\mathbf{z}}z$. Use of Eq. (4.21c) gives

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int_{l'} \hat{\mathbf{R}}'\,\frac{\rho_l\,dl'}{R'^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\int_{\phi=0}^{\pi/4} \rho_l \frac{(-\hat{\mathbf{x}}0.02\cos\phi - \hat{\mathbf{y}}0.02\sin\phi + \hat{\mathbf{z}}z)}{((0.02)^2 + z^2)^{3/2}}\,0.02\,d\phi$$

$$= \frac{898.8}{((0.02)^2 + z^2)^{3/2}}\left[-\hat{\mathbf{x}}0.014 - \hat{\mathbf{y}}0.006 + \hat{\mathbf{z}}0.78z\right] \qquad \text{(V/m)}.$$

(a) At $z = 0$, $\mathbf{E} = -\hat{\mathbf{x}}\,1.6 - \hat{\mathbf{y}}\,0.66$ (MV/m).

(b) At $z = 5$ cm, $\mathbf{E} = -\hat{\mathbf{x}}\,81.4 - \hat{\mathbf{y}}\,33.7 + \hat{\mathbf{z}}\,226$ (kV/m).

(c) At $z = -5$ cm, $\mathbf{E} = -\hat{\mathbf{x}}\,81.4 - \hat{\mathbf{y}}\,33.7 - \hat{\mathbf{z}}\,226$ (kV/m).

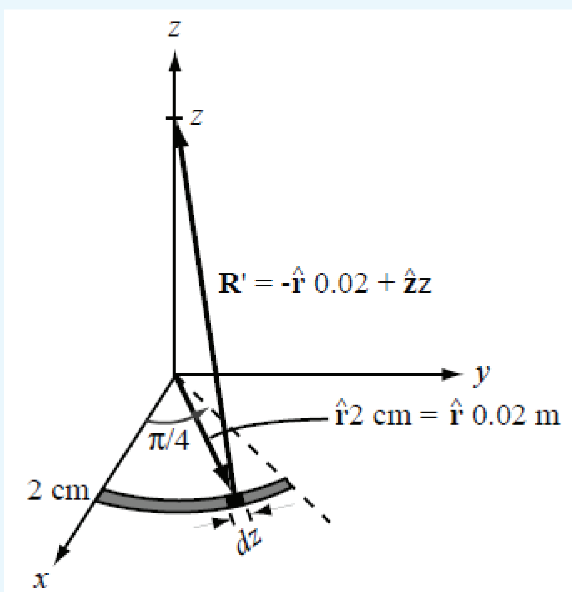

Figure P4.15 Line charge along an arc.