4.17 Repeat Example 4-5 for the circular disk of charge of radius a, but in the present case, assume the surface charge density to vary with r as

$$\rho_s = \rho_{s0} r^2 \qquad (\text{C/m}^2)$$

where ρ_{s0} is a constant.

Solution: We start with the expression for $d\mathbf{E}$ given in Example 4-5 but we replace ρ_s with $\rho_{s0} r^2$:

$$d\mathbf{E} = \hat{\mathbf{z}} \frac{h}{4\pi\varepsilon_0 (r^2+h^2)^{3/2}} (2\pi\rho_{s0} r^3\, dr),$$

$$\mathbf{E} = \hat{\mathbf{z}} \frac{\rho_{s0} h}{2\varepsilon_0} \int_0^a \frac{r^3\, dr}{(r^2+h^2)^{3/2}} .$$

To perform the integration, we use

$$\begin{aligned}
R^2 &= r^2 + h^2, \\
2R\, dR &= 2r\, dr, \\
\mathbf{E} &= \hat{\mathbf{z}} \frac{\rho_{s0} h}{2\varepsilon_0} \int_h^{(a^2+h^2)^{1/2}} \frac{(R^2-h^2)\, dR}{R^2} \\
&= \hat{\mathbf{z}} \frac{\rho_{s0} h}{2\varepsilon_0} \left[\int_h^{(a^2+h^2)^{1/2}} dR - \int_h^{(a^2+h^2)^{1/2}} \frac{h^2}{R^2}\, dR \right] \\
&= \hat{\mathbf{z}} \frac{\rho_{s0} h}{2\varepsilon_0} \left[\sqrt{a^2+h^2} + \frac{h^2}{\sqrt{a^2+h^2}} - 2h \right].
\end{aligned}$$