4.21 A horizontal strip lying in the x–y plane is of width d in the y-direction and infinitely long in the x-direction. If the strip is in air and has a uniform charge distribution ρ_s, use Coulomb's law to obtain an explicit expression for the electric field at a point P located at a distance h above the centerline of the strip. Extend your result to the special case where d is infinite and compare it with Eq. (4.25).

Solution:

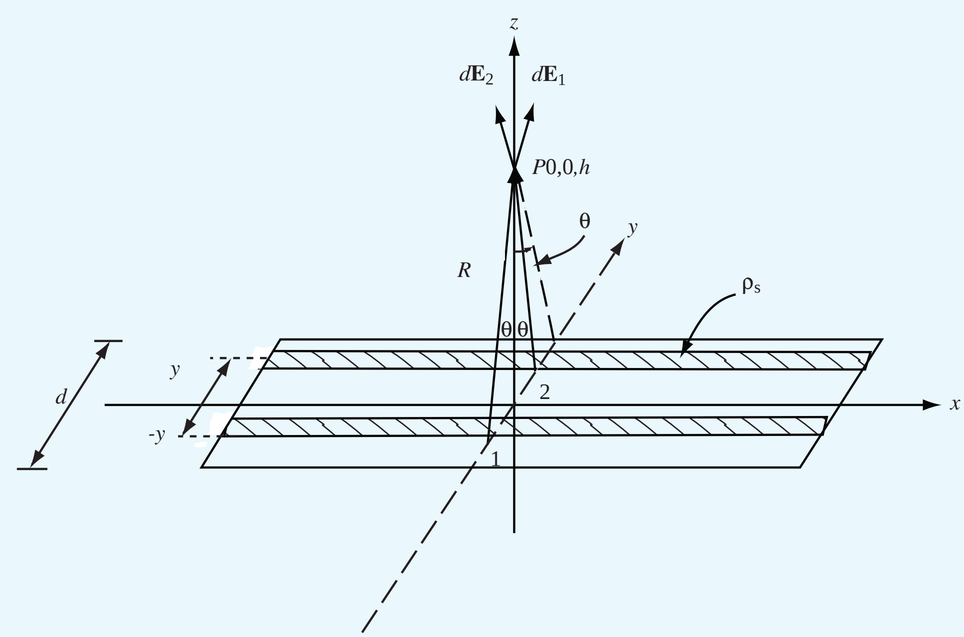

Figure P4.21 Horizontal strip of charge.

The strip of charge density ρ_s (C/m^2) can be treated as a set of adjacent line charges each of charge $\rho_l = \rho_s\, dy$ and width dy. At point P, the fields of line charge at distance y and line charge at distance $-y$ give contributions that cancel each other along $\hat{\mathbf{y}}$ and add along $\hat{\mathbf{z}}$. For each such pair,

$$d\mathbf{E} = \hat{\mathbf{z}}\,\frac{2\rho_s\, dy\cos\theta}{2\pi\varepsilon_0 R}\,.$$

With $R = h/\cos\theta$, we integrate from $y = 0$ to $d/2$, which corresponds to $\theta = 0$ to $\theta_0 = \sin^{-1}[(d/2)/(h^2 + (d/2)^2)^{1/2}]$. Thus,

$$\mathbf{E} = \int_0^{d/2} d\mathbf{E} = \hat{\mathbf{z}}\,\frac{\rho_s}{\pi\varepsilon_0}\int_0^{d/2}\frac{\cos\theta}{R}\,dy = \hat{\mathbf{z}}\,\frac{\rho_s}{\pi\varepsilon_0}\int_0^{\theta_0}\frac{\cos^2\theta}{h}\cdot\frac{h}{\cos^2\theta}\,d\theta$$

$$= \hat{\mathbf{z}} \frac{\rho_s}{\pi \varepsilon_0} \theta_0.$$

For an infinitely wide sheet, $\theta_0 = \pi/2$ and $\mathbf{E} = \hat{\mathbf{z}} \frac{\rho_s}{2\varepsilon_0}$, which is identical with Eq. (4.25).