4.26 In a certain region of space, the charge density is given in cylindrical coordinates by the function:

$$\rho_v = 5re^{-r} \quad (\text{C/m}^3)$$

Apply Gauss's law to find **D**.

Solution:

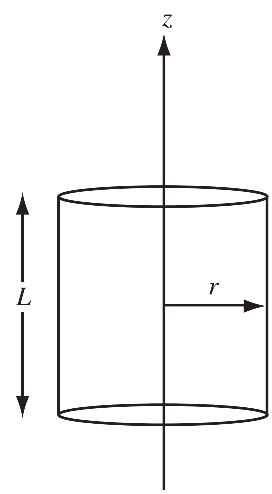

Figure P4.26 Gaussian surface.

Method 1: Integral Form of Gauss's Law

Since ρ_v varies as a function of r only, so will **D**. Hence, we construct a cylinder of radius r and length L, coincident with the z-axis. Symmetry suggests that **D** has the functional form $\mathbf{D} = \hat{\mathbf{r}} D$. Hence,

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q,$$

$$\int \hat{\mathbf{r}} D \cdot d\mathbf{s} = D(2\pi r L),$$

$$Q = 2\pi L \int_0^r 5re^{-r} \cdot r\, dr$$

$$= 10\pi L[-r^2 e^{-r} + 2(1 - e^{-r}(1+r))],$$

$$\mathbf{D} = \hat{\mathbf{r}} D = \hat{\mathbf{r}} 5 \left[\frac{2}{r}(1 - e^{-r}(1+r)) - re^{-r} \right].$$

Method 2: Differential Method

$$\nabla \cdot \mathbf{D} = \rho_{\mathrm{v}}, \qquad \mathbf{D} = \hat{\mathbf{r}} D_r,$$

with D_r being a function of r.

$$\frac{1}{r} \frac{\partial}{\partial r}(rD_r) = 5re^{-r},$$

$$\frac{\partial}{\partial r}(rD_r) = 5r^2 e^{-r},$$

$$\int_0^r \frac{\partial}{\partial r}(rD_r)\, dr = \int_0^r 5r^2 e^{-r}\, dr,$$

$$rD_r = 5[2(1 - e^{-r}(1+r)) - r^2 e^{-r}],$$

$$\mathbf{D} = \hat{\mathbf{r}}\, rD_r = \hat{\mathbf{r}}\, 5\left[\frac{2}{r}(1 - e^{-r}(1+r)) - re^{-r}\right].$$