4.28 If the charge density increases linearly with distance from the origin such that $\rho_v = 0$ at the origin and $\rho_v = 4$ C/m^3 at $R = 2$ m, find the corresponding variation of $\mathbf{D}$.

Solution:

$$\rho_v(R) = a + bR,$$
$$\rho_v(0) = a = 0,$$
$$\rho_v(2) = 2b = 4.$$

Hence, $b = 2$.

$$\rho_v(R) = 2R \qquad (\text{C/m}^3).$$

Applying Gauss's law to a spherical surface of radius R,

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = \int_V \rho_v \, d\mathcal{V},$$
$$D_R \cdot 4\pi R^2 = \int_0^R 2R \cdot 4\pi R^2 \, dR = 8\pi \frac{R^4}{4},$$
$$D_R = 0.5R^2 \qquad (\text{C/m}^2),$$
$$\mathbf{D} = \hat{\mathbf{R}} D_R = \hat{\mathbf{R}} 0.5R^2 \qquad (\text{C/m}^2).$$