4.29 A spherical shell with outer radius b surrounds a charge-free cavity of radius $a < b$ (Fig. P4.29). If the shell contains a charge density given by

$$\rho_v = -\frac{\rho_{v0}}{R^2}, \qquad a \le R \le b,$$

where ρ_{v0} is a positive constant, determine $\mathbf{D}$ in all regions.

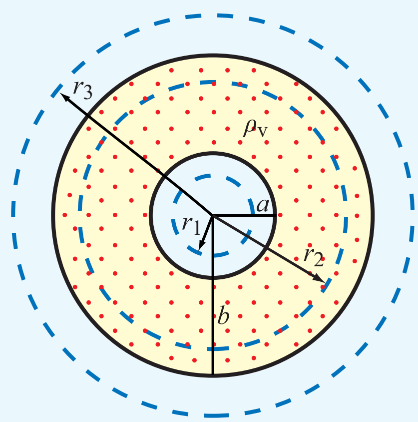

Figure P4.29 Problem 4.29.

Solution: Symmetry dictates that $\mathbf{D}$ is radially oriented. Thus,

$$\mathbf{D} = \hat{\mathbf{R}} D_R.$$

At any R, Gauss's law gives

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q$$

$$\int_S \hat{\mathbf{R}} D_R \cdot \hat{\mathbf{R}}\, ds = Q$$

$$4\pi R^2 D_R = Q$$

$$D_R = \frac{Q}{4\pi R^2}.$$

(a) For $R < a$, no charge is contained in the cavity. Hence, $Q = 0$, and

$$D_R = 0, \qquad R \le a.$$

(b) For $a \le R \le b$,

$$Q = \int_{R=a}^{R} \rho_{\rm v} \, dV = \int_{R=a}^{R} -\frac{\rho_{\rm v0}}{R^2} \cdot 4\pi R^2 \, dR$$
$$= -4\pi\rho_{\rm v0}(R-a).$$

Hence,

$$D_R = -\frac{\rho_{\rm v0}(R-a)}{R^2}, \qquad a \le R \le b.$$

(c) For $R \ge b$,

$$Q = \int_{R=a}^{b} \rho_{\rm v} \, dV = -4\pi\rho_{\rm v0}(b-a)$$

$$D_R = -\frac{\rho_{\rm v0}(b-a)}{R^2}, \qquad R \ge b.$$