4.31 The circular disk of radius a shown in Fig. 4-7 has uniform charge density ρ_s across its surface.

(a) Obtain an expression for the electric potential V at a point $P = (0, 0, z)$ on the z-axis.

(b) Use your result to find $\mathbf{E}$ and then evaluate it for $z = h$. Compare your final expression with (4.24), which was obtained on the basis of Coulomb's law.

Solution:

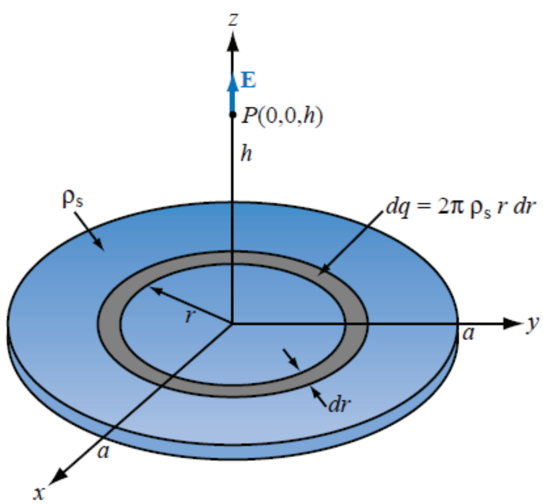

Figure P4.31 Circular disk of charge.

(a) Consider a ring of charge at a radial distance r. The charge contained in width dr is

$$dq = \rho_s(2\pi r\,dr) = 2\pi\rho_s r\,dr.$$

The potential at P is

$$dV = \frac{dq}{4\pi\varepsilon_0 R} = \frac{2\pi\rho_s r\,dr}{4\pi\varepsilon_0 (r^2+z^2)^{1/2}}.$$

The potential due to the entire disk is

$$V = \int_0^a dV = \frac{\rho_s}{2\varepsilon_0}\int_0^a \frac{r\,dr}{(r^2+z^2)^{1/2}} = \frac{\rho_s}{2\varepsilon_0}\,(r^2+z^2)^{1/2}\bigg|_0^a = \frac{\rho_s}{2\varepsilon_0}\left[(a^2+z^2)^{1/2} - z\right].$$

(b)

$$\mathbf{E} = -\nabla V = -\hat{\mathbf{x}}\,\frac{\partial V}{\partial x} - \hat{\mathbf{y}}\,\frac{\partial V}{\partial y} - \hat{\mathbf{z}}\,\frac{\partial V}{\partial z} = \hat{\mathbf{z}}\,\frac{\rho_{\mathrm{s}}}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{a^2 + z^2}}\right].$$

The expression for $\mathbf{E}$ reduces to Eq. (4.24) when $z = h$.