

4.40 The x - y plane contains a uniform sheet of charge with $\rho_{s1} = 0.2$ (nC/m²). A second sheet with $\rho_{s2} = -0.2$ (nC/m²) occupies the plane $z = 6$ m. Find V_{AB} , V_{BC} , and V_{AC} for $A(0, 0, 6$ m), $B(0, 0, 0)$, and $C(0, -2$ m, 2 m).

Solution: We start by finding the \mathbf{E} field in the region between the plates. For any point above the x - y plane, \mathbf{E}_1 due to the charge on x - y plane is, from Eq. (4.25),

$$\mathbf{E}_1 = \hat{\mathbf{z}} \frac{\rho_{s1}}{2\epsilon_0}.$$

In the region below the top plate, \mathbf{E} would point downwards for positive ρ_{s2} on the top plate. In this case, $\rho_{s2} = -\rho_{s1}$. Hence,

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \hat{\mathbf{z}} \frac{\rho_{s1}}{2\epsilon_0} - \hat{\mathbf{z}} \frac{\rho_{s2}}{2\epsilon_0} = \hat{\mathbf{z}} \frac{2\rho_{s1}}{2\epsilon_0} = \hat{\mathbf{z}} \frac{\rho_{s1}}{\epsilon_0}.$$

Since \mathbf{E} is along $\hat{\mathbf{z}}$, only change in position along z can result in change in voltage.

$$V_{AB} = - \int_0^6 \hat{\mathbf{z}} \frac{\rho_{s1}}{\epsilon_0} \cdot \hat{\mathbf{z}} dz = - \frac{\rho_{s1}}{\epsilon_0} z \Big|_0^6 = - \frac{6\rho_{s1}}{\epsilon_0} = - \frac{6 \times 0.2 \times 10^{-9}}{8.85 \times 10^{-12}} = -135.59 \text{ V}.$$

The voltage at C depends only on the z -coordinate of C . Hence, with point A being at the lowest potential and B at the highest potential,

$$V_{BC} = \frac{-2}{6} V_{AB} = - \frac{(-135.59)}{3} = 45.20 \text{ V},$$

$$V_{AC} = V_{AB} + V_{BC} = -135.59 + 45.20 = -90.39 \text{ V}.$$

