4.47 A cylinder-shaped carbon resistor is 8 cm in length and its circular cross section has a diameter $d = 1$ mm.

(a) Determine the resistance R.

(b) To reduce its resistance by 40%, the carbon resistor is coated with a layer of copper of thickness t. Use the result of Problem 4.44 to determine t.

Solution:

(a) From (4.70), and using the value of σ for carbon from Appendix B,

$$R = \frac{l}{\sigma A} = \frac{l}{\sigma\pi(d/2)^2} = \frac{8\times 10^{-2}}{3\times 10^4\pi(10^{-3}/2)^2} = 3.4\ \Omega.$$

(b) The 40%-reduced resistance is:

$$R' = 0.6R = 0.6\times 3.4 = 2.04\ \Omega.$$

Using the result of Problem 4.40:

$$R' = \frac{l}{\pi(\sigma_1 a^2 + \sigma_2(b^2 - a^2))} = 2.04\ \Omega.$$

With $\sigma_1 = 3.4\times 10^4$ S/m (carbon), $\sigma_2 = 5.8\times 10^7$ S/m (copper), $a = 1$ mm$/2 = 5\times 10^{-4}$ m, and b unknown, we have

$$b = 5.00086\times 10^{-4}\ \text{m}$$

and

$$\begin{aligned} t = b - a &= (5.00086 - 5)\times 10^{-4} \\ &= 0.00086\times 10^{-4}\ \text{m} = 0.086\ \mu\text{m}. \end{aligned}$$

Thus, the addition of a copper coating less than 0.1 μm in thickness reduces the resistance by 40%.