

4.49 An infinitely long conducting cylinder of radius a is surrounded by a dielectric medium that contains no free charges. If the tangential component of the electric field in the region $r \geq a$ is given by $\mathbf{E}_t = -\hat{\phi} \cos \phi / r^2$, find \mathbf{E} in that region.

Solution: Let the conducting cylinder be medium 1 and the surrounding dielectric medium be medium 2. In medium 2,

$$\mathbf{E}_2 = \hat{\mathbf{r}} E_r - \hat{\phi} \frac{1}{r^2} \cos \phi,$$

with E_r , the normal component of \mathbf{E}_2 , unknown. We invoke Gauss's law in medium 2:

$$\nabla \cdot \mathbf{D}_2 = 0,$$

or

$$\frac{1}{r} \frac{\partial}{\partial r} (r E_r) + \frac{1}{r} \frac{\partial}{\partial \phi} \left(-\frac{1}{r^2} \cos \phi \right) = 0,$$

which leads to

$$\frac{\partial}{\partial r} (r E_r) = \frac{\partial}{\partial \phi} \left(\frac{1}{r^2} \cos \phi \right) = -\frac{1}{r^2} \sin \phi.$$

Integrating both sides with respect to r ,

$$\begin{aligned} \int \frac{\partial}{\partial r} (r E_r) dr &= -\sin \phi \int \frac{1}{r^2} dr \\ r E_r &= \frac{1}{r} \sin \phi, \end{aligned}$$

or

$$E_r = \frac{1}{r^2} \sin \phi.$$

Hence,

$$\mathbf{E}_2 = \hat{\mathbf{r}} \frac{1}{r^2} \sin \phi - \hat{\phi} \frac{1}{r^2} \cos \phi.$$
