**4.63** Use the image method to find the capacitance per unit length of an infinitely long conducting cylinder of radius $a$ situated at a distance $d$ from a parallel conducting plane, as shown in Fig. P4.63.

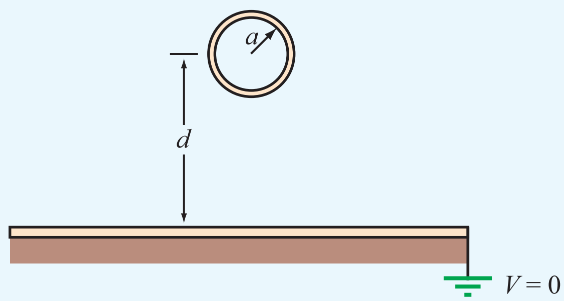


**Figure P4.63** Conducting cylinder above a conducting plane (Problem 4.63).

**Solution:** Let us distribute charge $\rho_l$ (C/m) on the conducting cylinder. Its image cylinder at $z = -d$ will have charge density $-\rho_l$.

For the line at $z = d$, the electric field at any point $z$ (at a distance of $d - z$ from the center of the cylinder) is, from Eq. (4.33),

$$\mathbf{E}_1 = -\hat{\mathbf{z}} \frac{\rho_l}{2\pi\varepsilon_0(d-z)}$$

where $-\hat{\mathbf{z}}$ is the direction away from the cylinder. Similarly for the image cylinder at distance $(d+z)$ and carrying charge $-\rho_l$,

$$\mathbf{E}_2 = \hat{\mathbf{z}} \frac{(-\rho_l)}{2\pi\varepsilon_0(d+z)} = -\hat{\mathbf{z}} \frac{\rho_l}{2\pi\varepsilon_0(d+z)}.$$

The potential difference between the cylinders is obtained by integrating the total electric field from $z = -(d-a)$ to $z = (d-a)$:

$$\begin{aligned}
V &= -\int_2^1 (\mathbf{E}_1 + \mathbf{E}_2) \cdot \hat{\mathbf{z}}\, dz \\
&= -\int_{-(d-a)}^{d-a} -\hat{\mathbf{z}} \frac{\rho_l}{2\pi\varepsilon_0} \left( \frac{1}{d-z} + \frac{1}{d+z} \right) \cdot \hat{\mathbf{z}}\, dz \\
&= \frac{\rho_l}{2\pi\varepsilon_0} \int_{-(d-a)}^{d-a} \left( \frac{1}{d-z} + \frac{1}{d+z} \right) dz \\
&= \frac{\rho_l}{2\pi\varepsilon_0} \left[ -\ln(d-z) + \ln(d+z) \right]_{-(d-a)}^{d-a}
\end{aligned}$$

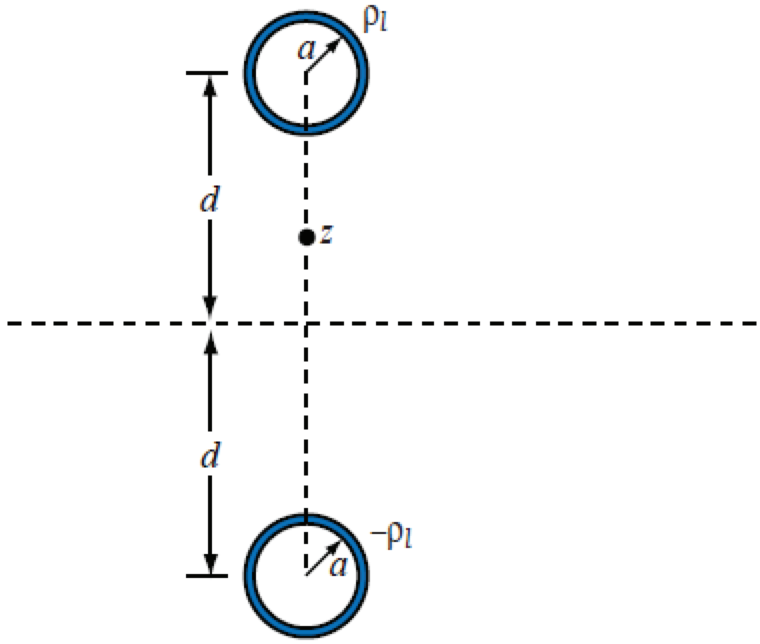


**Figure P4.63** (a) Cylinder and its image.

$$= \frac{\rho_l}{2\pi\varepsilon_0} [-\ln(a) + \ln(2d-a) + \ln(2d-a) - \ln(a)]$$
$$= \frac{\rho_l}{\pi\varepsilon_0} \ln\left(\frac{2d-a}{a}\right).$$

For a length $L$, $Q = \rho_l L$ and

$$C = \frac{Q}{V} = \frac{\rho_l L}{(\rho_l/\pi\varepsilon_0)\ln[(2d-a)/a]},$$

and the capacitance per unit length is

$$C' = \frac{C}{L} = \frac{\pi\varepsilon_0}{\ln[(2d/a)-1]} \qquad \text{(C/m)}.$$