

**4.8** An electron beam shaped like a circular cylinder of radius  $r_0$  carries a charge density given by

$$\rho_v = \left( \frac{-\rho_0}{1+r^2} \right) \quad (\text{C/m}^3)$$

where  $\rho_0$  is a positive constant and the beam's axis is coincident with the  $z$ -axis.

- (a) Determine the total charge contained in length  $L$  of the beam.
- (b) If the electrons are moving in the  $+z$ -direction with uniform speed  $u$ , determine the magnitude and direction of the current crossing the  $z$ -plane.

**Solution:**

(a)

$$\begin{aligned} Q &= \int_{r=0}^{r_0} \int_{z=0}^L \rho_v \, d\mathcal{V} = \int_{r=0}^{r_0} \int_{z=0}^L \left( \frac{-\rho_0}{1+r^2} \right) 2\pi r \, dr \, dz \\ &= -2\pi\rho_0 L \int_0^{r_0} \frac{r}{1+r^2} \, dr = -\pi\rho_0 L \ln(1+r_0^2). \end{aligned}$$

(b)

$$\begin{aligned} \mathbf{J} &= \rho_v \mathbf{u} = -\hat{\mathbf{z}} \frac{u\rho_0}{1+r^2} \quad (\text{A/m}^2), \\ I &= \int \mathbf{J} \cdot d\mathbf{s} \\ &= \int_{r=0}^{r_0} \int_{\phi=0}^{2\pi} \left( -\hat{\mathbf{z}} \frac{u\rho_0}{1+r^2} \right) \cdot \hat{\mathbf{z}} r \, dr \, d\phi \\ &= -2\pi u\rho_0 \int_0^{r_0} \frac{r}{1+r^2} \, dr = -\pi u\rho_0 \ln(1+r_0^2) \quad (\text{A}). \end{aligned}$$

Current direction is along  $-\hat{\mathbf{z}}$ .

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