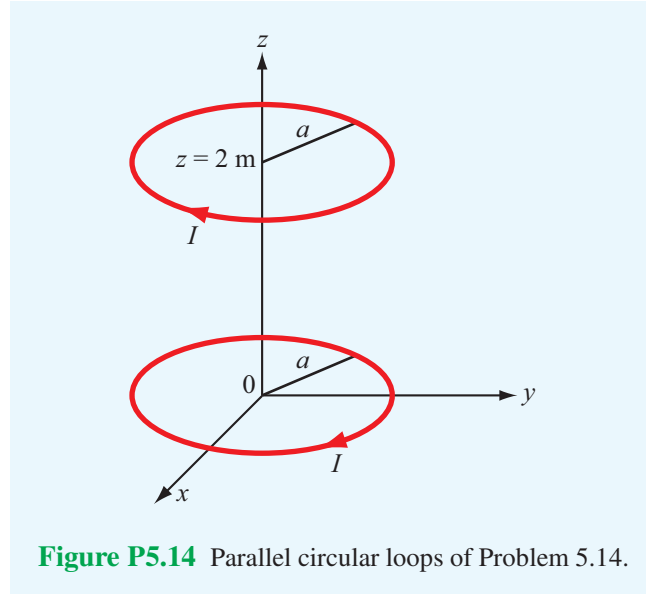


**5.14** Two parallel, circular loops carrying a current of 40 A each are arranged as shown in Fig. P5.14. The first loop is situated in the  $x$ - $y$  plane with its center at the origin, and the second loop's center is at  $z = 2$  m. If the two loops have the same radius  $a = 3$  m, determine the magnetic field at:

- (a)  $z = 0$
- (b)  $z = 1$  m
- (c)  $z = 2$  m



**Solution:** The magnetic field due to a circular loop is given by (5.34) for a loop in the  $x$ - $y$  plane carrying a current  $I$  in the  $+\hat{\phi}$  direction. Considering that the bottom loop in Fig. is in the  $x$ - $y$  plane, but the current direction is along  $-\hat{\phi}$ ,

$$\mathbf{H}_1 = -\hat{\mathbf{z}} \frac{Ia^2}{2(a^2 + z^2)^{3/2}},$$

where  $z$  is the observation point along the  $z$ -axis. For the second loop, which is at a height of 2 m, we can use the same expression but  $z$  should be replaced with  $(z - 2)$ . Hence,

$$\mathbf{H}_2 = -\hat{\mathbf{z}} \frac{Ia^2}{2[a^2 + (z - 2)^2]^{3/2}}.$$

The total field is

$$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2 = -\hat{\mathbf{z}} \frac{Ia^2}{2} \left[ \frac{1}{(a^2 + z^2)^{3/2}} + \frac{1}{[a^2 + (z - 2)^2]^{3/2}} \right] \text{ A/m.}$$

(a) At  $z = 0$ , and with  $a = 3$  m and  $I = 40$  A,

$$\mathbf{H} = -\hat{\mathbf{z}} \frac{40 \times 9}{2} \left[ \frac{1}{3^3} + \frac{1}{(9+4)^{3/2}} \right] = -\hat{\mathbf{z}} 10.5 \text{ A/m.}$$

(b) At  $z = 1$  m (midway between the loops):

$$\mathbf{H} = -\hat{\mathbf{z}} \frac{40 \times 9}{2} \left[ \frac{1}{(9+1)^{3/2}} + \frac{1}{(9+1)^{3/2}} \right] = -\hat{\mathbf{z}} 11.38 \text{ A/m.}$$

(c) At  $z = 2$  m,  $\mathbf{H}$  should be the same as at  $z = 0$ . Thus,

$$\mathbf{H} = -\hat{\mathbf{z}} 10.5 \text{ A/m.}$$

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