5.19 Three long, parallel wires are arranged as shown in Fig. P5.19. Determine the force per unit length acting on the wire carrying I_3.

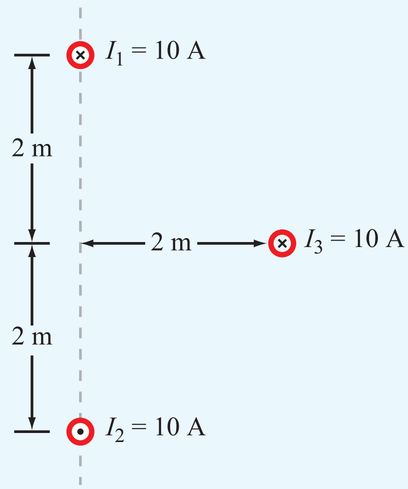

Figure P5.19 Three parallel wires of Problem 5.19.

Solution: Since I_1 and I_2 are equal in magnitude and opposite in direction, and equidistant from I_3, our intuitive answer might be that the net force on I_3 is zero. As we will see, that's not the correct answer. The field due to I_1 (which is along $\hat{\mathbf{y}}$) at location of I_3 is

$$\mathbf{B}_1 = \hat{\mathbf{b}}_1 \frac{\mu_0 I_1}{2\pi R_1}$$

where $\hat{\mathbf{b}}_1$ is the unit vector in the direction of $\mathbf{B}_1$ shown in the figure, which is perpendicular to $\hat{\mathbf{R}}_1$. The force per unit length exerted on I_3 is

$$\mathbf{F}'_{31} = \frac{\mu_0 I_1 I_3}{2\pi R_1} (\hat{\mathbf{y}} \times \hat{\mathbf{b}}_1) = -\hat{\mathbf{R}}_1 \frac{\mu_0 I_1 I_3}{2\pi R_1}.$$

Similarly, the force per unit length excited on I_3 by the field due to I_2 (which is along $-\hat{\mathbf{y}}$) is

$$\mathbf{F}'_{32} = \hat{\mathbf{R}}_2 \frac{\mu_0 I_2 I_3}{2\pi R_2}.$$

The two forces have opposite components along $\hat{\mathbf{x}}$ and equal components along $\hat{\mathbf{z}}$. Hence, with $R_1 = R_2 = \sqrt{8}$ m and $\theta = \sin^{-1}(2/\sqrt{8}) = \sin^{-1}(1/\sqrt{2}) = 45°$,

$$\mathbf{F}'_3 = \mathbf{F}'_{31} + \mathbf{F}'_{32} = \hat{\mathbf{z}} \left(\frac{\mu_0 I_1 I_3}{2\pi R_1} + \frac{\mu_0 I_2 I_3}{2\pi R_2} \right) \sin\theta$$

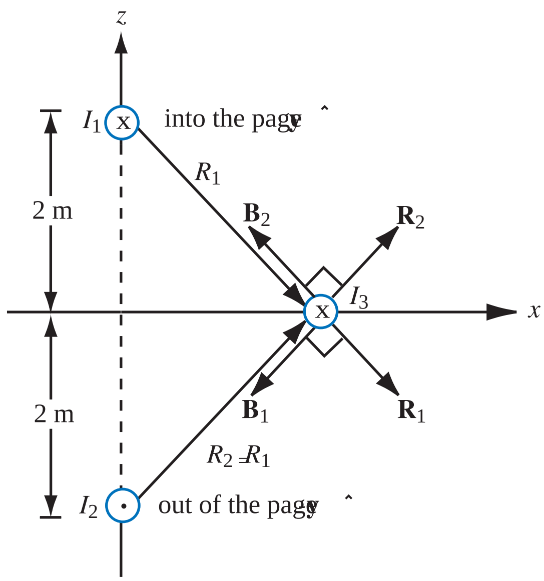

Figure P5.19 (a) **B** fields due to I_1 and I_2 at location of I_3.

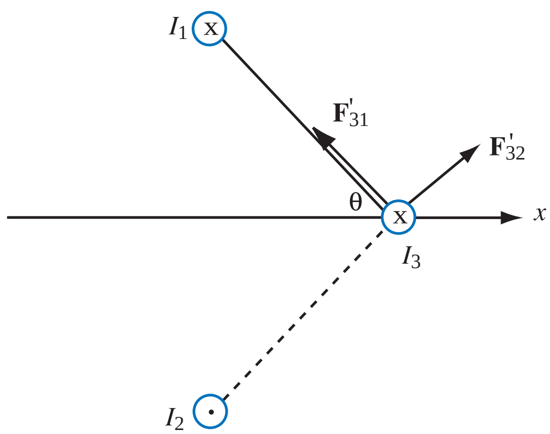

Figure P5.19 (b) Forces acting on I_3.

$$= \hat{\mathbf{z}}2\left(\frac{4\pi \times 10^{-7} \times 10 \times 20}{2\pi \times \sqrt{8}}\right) \times \frac{1}{\sqrt{2}} = \hat{\mathbf{z}}2 \times 10^{-5} \text{ N/m.}$$