5.3 The circuit shown in Fig. P5.3 uses two identical springs to support a 10-cm-long horizontal wire with a mass of 20 g. In the absence of a magnetic field, the weight of the wire causes the springs to stretch a distance of 0.2 cm each. When a uniform magnetic field is turned on in the region containing the horizontal wire, the springs are observed to stretch an additional 0.5 cm each. What is the intensity of the magnetic flux density **B**? The force equation for a spring is $F = kd$, where k is the spring constant and d is the distance it has been stretched.

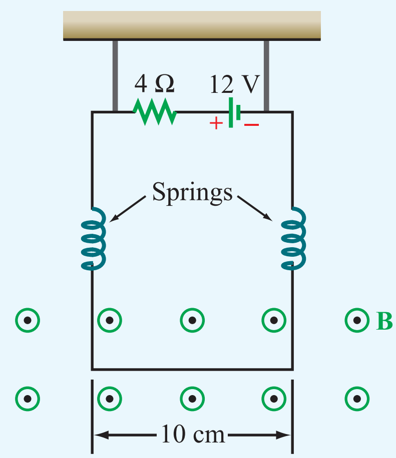

Figure P5.3 Configuration of Problem 5.3.

Solution: Springs are characterized by a spring constant k where $F = kd$ is the force exerted on the spring and d is the amount the spring is stretched from its rest configuration. In this instance, each spring sees half the weight of the wire:

$$F = \tfrac{1}{2}mg = kd, \qquad k = \frac{mg}{2d} = \frac{20\times10^{-3}\times9.8}{2\times2\times10^{-3}} = 49 \qquad \text{(N/m)}.$$

Therefore, when the springs are further stretched by an additional 0.5 cm, this amounts to an additional force of $F = 49\text{ N/m}\times\left(5\times10^{-3}\text{ m}\right) = 245$ mN per spring, or a total additional force of $F = 0.49$ N. This force is equal to the force exerted on the wire by the interaction of the magnetic field and the current as described by Eq. (5.12): $\vec{F}_{\text{m}} = I\boldsymbol{\ell}\times\vec{B}$, where $\boldsymbol{\ell}$ and $\vec{B}$ are at right angles. Moreover $\boldsymbol{\ell}\times\vec{B}$ is in the downward direction, and $I = V/R = 12\text{ V}/4\ \Omega = 3$ A. Therefore,

$$\left|\vec{F}_{\text{m}}\right| = |I||\boldsymbol{\ell}|\left|\vec{B}\right|, \qquad \left|\vec{B}\right| = \frac{\left|\vec{F}_{\text{m}}\right|}{|I||\boldsymbol{\ell}|} = \frac{0.49}{3\times0.1} = 1.63 \qquad \text{(T)}.$$