5.40 The rectangular loop shown in Fig. P5.40 is coplanar with the long, straight wire carrying the current $I = 20$ A. Determine the magnetic flux through the loop.

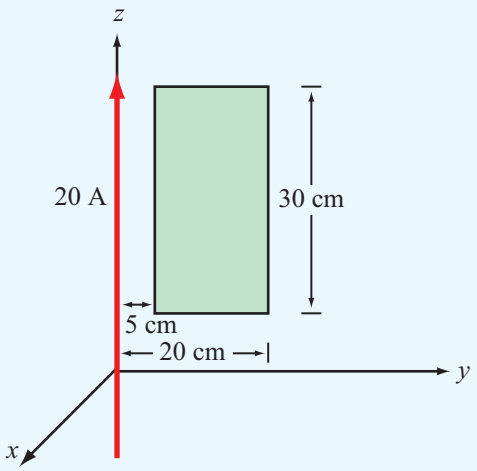

Figure P5.40 Loop and wire arrangement for Problem 5.40.

Solution: The field due to the long wire is, from Eq. (5.30),

$$\mathbf{B} = \hat{\boldsymbol{\phi}}\frac{\mu_0 I}{2\pi r} = -\hat{\mathbf{x}}\frac{\mu_0 I}{2\pi r} = -\hat{\mathbf{x}}\frac{\mu_0 I}{2\pi y},$$

where in the plane of the loop, $\hat{\boldsymbol{\phi}}$ becomes $-\hat{\mathbf{x}}$ and r becomes y.

The flux through the loop is along $-\hat{\mathbf{x}}$, and the magnitude of the flux is

$$\begin{aligned}\Phi = \int_S \mathbf{B}\cdot d\mathbf{s} &= \frac{\mu_0 I}{2\pi}\int_{5\text{ cm}}^{20\text{ cm}} -\frac{\hat{\mathbf{x}}}{y}\cdot -\hat{\mathbf{x}}\,(30\text{ cm}\times dy)\\ &= \frac{\mu_0 I}{2\pi}\times 0.3\int_{0.05}^{0.2}\frac{dy}{y}\\ &= \frac{0.3\,\mu_0}{2\pi}\times 20\times \ln\left(\frac{0.2}{0.05}\right) = 1.66\times 10^{-6} \qquad \text{(Wb)}.\end{aligned}$$