5.7 An 8 cm $\times$ 12 cm rectangular loop of wire is situated in the x–y plane with the center of the loop at the origin and its long sides parallel to the x-axis. The loop has a current of 50 A flowing clockwise (when viewed from above). Determine the magnetic field at the center of the loop.

Solution: The total magnetic field is the vector sum of the individual fields of each of the four wire segments: $\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4$. An expression for the magnetic field from a wire segment is given by Eq. (5.29). For all segments shown in Fig. P5.7, the

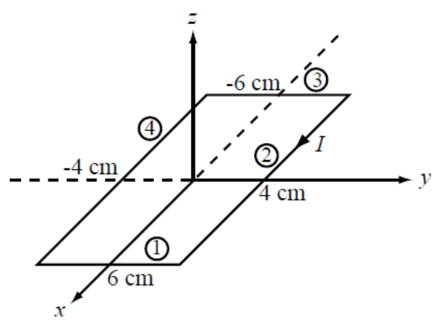

Figure P5.7 Problem 5.7.

combination of the direction of the current and the right-hand rule gives the direction of the magnetic field as $-z$ direction at the origin. With $r = 6$ cm and $l = 8$ cm,

$$\vec{B}_1 = -\hat{\mathbf{z}}\frac{\mu Il}{2\pi r\sqrt{4r^2+l^2}}$$

$$= -\hat{\mathbf{z}}\frac{4\pi \times 10^{-7} \times 50 \times 0.08}{2\pi \times 0.06 \times \sqrt{4 \times 0.06^2 + 0.08^2}} = -\hat{\mathbf{z}}9.24 \times 10^{-5} \qquad \text{(T)}.$$

For segment 2, $r = 4$ cm and $l = 12$ cm,

$$\mathbf{B}_2 = -\hat{\mathbf{z}}\frac{\mu Il}{2\pi r\sqrt{4r^2+l^2}}$$

$$= -\hat{\mathbf{z}}\frac{4\pi \times 10^{-7} \times 50 \times 0.12}{2\pi \times 0.04 \times \sqrt{4 \times 0.04^2 + 0.12^2}} = -\hat{\mathbf{z}}20.80 \times 10^{-5} \qquad \text{(T)}.$$

Similarly,

$$\vec{B}_3 = -\hat{\mathbf{z}}9.24 \times 10^{-5} \quad \text{(T)}, \qquad \vec{B}_4 = -\hat{\mathbf{z}}20.80 \times 10^{-5} \quad \text{(T)}.$$

The total field is then $\vec{B} = \mathbf{B}_1 + \mathbf{B}_2 + \mathbf{B}_3 + \mathbf{B}_4 = -\hat{\mathbf{z}}0.60$ (mT).