

6.15 A coaxial capacitor of length $l = 6$ cm uses an insulating dielectric material with $\epsilon_r = 9$. The radii of the cylindrical conductors are 0.5 cm and 1 cm. If the voltage applied across the capacitor is

$$V(t) = 50 \sin(120\pi t) \quad (\text{V})$$

what is the displacement current?

Solution:

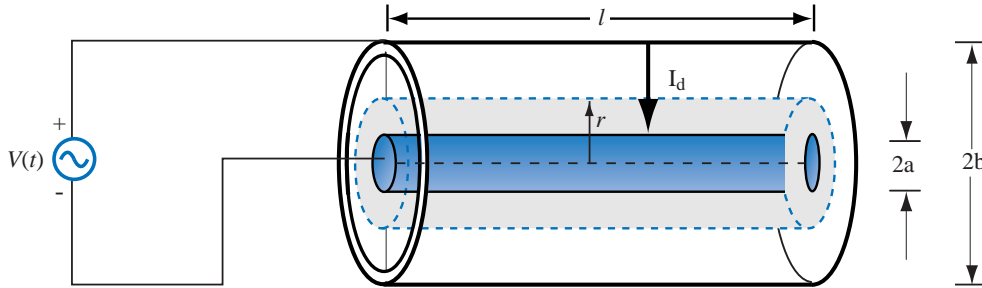


Figure P6.15

To find the displacement current, we need to know \mathbf{E} in the dielectric space between the cylindrical conductors. From Eqs. (4.114) and (4.115),

$$\mathbf{E} = -\hat{\mathbf{r}} \frac{Q}{2\pi\epsilon r l},$$

$$V = \frac{Q}{2\pi\epsilon l} \ln\left(\frac{b}{a}\right).$$

Hence,

$$\mathbf{E} = -\hat{\mathbf{r}} \frac{V}{r \ln\left(\frac{b}{a}\right)} = -\hat{\mathbf{r}} \frac{50 \sin(120\pi t)}{r \ln 2} = -\hat{\mathbf{r}} \frac{72.1}{r} \sin(120\pi t) \quad (\text{V/m}),$$

$$\mathbf{D} = \epsilon \mathbf{E}$$

$$= \epsilon_r \epsilon_0 \mathbf{E}$$

$$= -\hat{\mathbf{r}} 9 \times 8.85 \times 10^{-12} \times \frac{72.1}{r} \sin(120\pi t)$$

$$= -\hat{\mathbf{r}} \frac{5.75 \times 10^{-9}}{r} \sin(120\pi t) \quad (\text{C/m}^2).$$

The displacement current flows between the conductors through an imaginary cylindrical surface of length l and radius r . The current flowing from the outer

conductor to the inner conductor along $-\hat{\mathbf{r}}$ crosses surface \mathbf{S} where

$$\mathbf{S} = -\hat{\mathbf{r}} 2\pi r l.$$

Hence,

$$\begin{aligned} I_d &= \frac{\partial \mathbf{D}}{\partial t} \cdot \mathbf{S} = -\hat{\mathbf{r}} \frac{\partial}{\partial t} \left(\frac{5.75 \times 10^{-9}}{r} \sin(120\pi t) \right) \cdot (-\hat{\mathbf{r}} 2\pi r l) \\ &= 5.75 \times 10^{-9} \times 120\pi \times 2\pi l \cos(120\pi t) \\ &= 0.82 \cos(120\pi t) \quad (\mu\text{A}). \end{aligned}$$

Alternatively, since the coaxial capacitor is lossless, its displacement current has to be equal to the conduction current flowing through the wires connected to the voltage sources. The capacitance of a coaxial capacitor is given by (4.116) as

$$C = \frac{2\pi\epsilon l}{\ln\left(\frac{b}{a}\right)}.$$

The current is

$$I = C \frac{dV}{dt} = \frac{2\pi\epsilon l}{\ln\left(\frac{b}{a}\right)} [120\pi \times 50 \cos(120\pi t)] = 0.82 \cos(120\pi t) \quad (\mu\text{A}),$$

which is the same answer we obtained before.
