6.24 The magnetic field in a dielectric material with $\varepsilon = 4\varepsilon_0$, $\mu = \mu_0$, and $\sigma = 0$ is given by

$$\mathbf{H}(y,t) = \hat{\mathbf{x}}5\cos(2\pi \times 10^7 t + ky) \qquad \text{(A/m)}.$$

Find k and the associated electric field $\mathbf{E}$.

Solution: In phasor form, the magnetic field is given by $\widetilde{\vec{H}} = \hat{\mathbf{x}}5e^{jky}$ (A/m). From Eq. (6.86),

$$\widetilde{\vec{E}} = \frac{1}{j\omega\varepsilon}\nabla\times\widetilde{\vec{H}} = \frac{-jk}{j\omega\varepsilon}\hat{\mathbf{z}}5\exp jky$$

and, from Eq. (6.87),

$$\widetilde{\vec{H}} = \frac{1}{-j\omega\mu}\nabla\times\widetilde{\vec{E}} = \frac{-jk^2}{-j\omega^2\varepsilon\mu}\hat{\mathbf{x}}5\exp jky,$$

which, together with the original phasor expression for $\widetilde{\vec{H}}$, implies that

$$k = \omega\sqrt{\varepsilon\mu} = \frac{\omega\sqrt{\varepsilon_r}}{c} = \frac{2\pi \times 10^7\sqrt{4}}{3\times 10^8} = \frac{4\pi}{30} \qquad \text{(rad/m)}.$$

Inserting this value in the expression for $\widetilde{\vec{E}}$ above,

$$\widetilde{\vec{E}} = -\hat{\mathbf{z}}\frac{4\pi/30}{2\pi\times 10^7\times 4\times 8.854\times 10^{-12}}5\exp j4\pi y/30 = -\hat{\mathbf{z}}941\exp j4\pi y/30 \qquad \text{(V/m)}.$$