6.26 The electric field radiated by a short dipole antenna is given in spherical coordinates by

$$\mathbf{E}(R,\theta;t) = \hat{\boldsymbol{\theta}}\,\frac{2\times10^{-2}}{R}\sin\theta\,\cos(6\pi\times10^{8}t-2\pi R)\quad(\text{V/m}).$$

Find $\mathbf{H}(R,\theta;t)$.

Solution: Converting to phasor form, the electric field is given by

$$\tilde{\vec{E}}(R,\theta) = \hat{\boldsymbol{\theta}}E_\theta = \hat{\boldsymbol{\theta}}\frac{2\times10^{-2}}{R}\sin\theta\exp-j2\pi R \qquad (\text{V/m}),$$

which can be used with Eq. (6.87) to find the magnetic field:

$$\begin{aligned}
\tilde{\vec{H}}(R,\theta) &= \frac{1}{-j\omega\mu}\nabla\times\tilde{\vec{E}} = \frac{1}{-j\omega\mu}\left[\hat{\mathbf{R}}\frac{1}{R\sin\theta}\frac{\partial E_\theta}{\partial\phi}+\hat{\boldsymbol{\phi}}\frac{1}{R}\frac{\partial}{\partial R}(RE_\theta)\right]\\
&= \frac{1}{-j\omega\mu}\hat{\boldsymbol{\phi}}\frac{2\times10^{-2}}{R}\sin\theta\,\frac{\partial}{\partial R}(\exp-j2\pi R)\\
&= \hat{\boldsymbol{\phi}}\frac{2\pi}{6\pi\times10^{8}\times4\pi\times10^{-7}}\frac{2\times10^{-2}}{R}\sin\theta\exp-j2\pi R\\
&= \hat{\boldsymbol{\phi}}\frac{53}{R}\sin\theta\;\exp-j2\pi R \qquad (\mu\text{A/m}).
\end{aligned}$$

Converting back to instantaneous value, this is

$$\vec{H}(R,\theta;t) = \hat{\boldsymbol{\phi}}\frac{53}{R}\sin\theta\cos\left(6\pi\times10^{8}t-2\pi R\right) \qquad (\mu\text{A/m}).$$