6.3 A coil consists of 100 turns of wire wrapped around a square frame of sides 0.25 m. The coil is centered at the origin with each of its sides parallel to the x- or y-axis. Find the induced emf across the open-circuited ends of the coil if the magnetic field is given by

(a) $\mathbf{B} = \hat{\mathbf{z}}\, 20e^{-3t}$ (T)

(b) $\mathbf{B} = \hat{\mathbf{z}}\, 20\cos x\ \cos 10^3 t$ (T)

(c) $\mathbf{B} = \hat{\mathbf{z}}\, 20\cos x\ \sin 2y\ \cos 10^3 t$ (T)

Solution: Since the coil is not moving or changing shape, $V_{\text{emf}}^{\text{m}} = 0$ V and $V_{\text{emf}} = V_{\text{emf}}^{\text{tr}}$. From Eq. (6.6),

$$V_{\text{emf}} = -N\frac{d}{dt}\int_S \vec{B}\cdot d\vec{s} = -N\frac{d}{dt}\int_{-0.125}^{0.125}\int_{-0.125}^{0.125} \vec{B}\cdot(\hat{\mathbf{z}}\, dx\, dy)\,,$$

where $N = 100$ and the surface normal was chosen to be in the $+\hat{\mathbf{z}}$ direction.

(a) For $\vec{B} = \hat{\mathbf{z}}20e^{-3t}$ (T),

$$V_{\text{emf}} = -100\frac{d}{dt}\left(20\exp -3t\,(0.25)^2\right) = 375e^{-3t} \qquad \text{(V)}.$$

(b) For $\vec{B} = \hat{\mathbf{z}}20\cos x \cos 10^3 t$ (T),

$$V_{\text{emf}} = -100\frac{d}{dt}\left(20\cos 10^3 t \int_{x=-0.125}^{0.125}\int_{y=-0.125}^{0.125} \cos x\, dx\, dy\right) = 124.6\sin 10^3 t \qquad \text{(kV)}.$$

(c) For $\vec{B} = \hat{\mathbf{z}}20\cos x \sin 2y \cos 10^3 t$ (T),

$$V_{\text{emf}} = -100\frac{d}{dt}\left(20\cos 10^3 t \int_{x=-0.125}^{0.125}\int_{y=-0.125}^{0.125} \cos x \sin 2y\, dx\, dy\right) = 0.$$