7.12 The magnetic field of a uniform plane wave propagating in a dielectric medium with $\varepsilon_r = 36$ is given by

$$\widetilde{\mathbf{H}} = 30(\hat{\mathbf{y}} + j\hat{\mathbf{z}})e^{-j\pi x/6} \qquad \text{(mA/m)}.$$

Specify the modulus and direction of the electric field intensity at the $x = 0$ plane at $t = 0$ and 5 ns.

Solution: From the expression for $\widetilde{\mathbf{H}}$, we deduce that the wave is traveling in the $+x$ direction and that

$$k = \frac{\pi}{6} \quad \text{rad/m}.$$

Hence,

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi} \times 6 = 12 \text{ m}.$$

Also,

$$u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{36}} = 0.5 \times 10^8 \text{ m/s}.$$

Hence,

$$f = \frac{u_p}{\lambda} = \frac{0.5 \times 10^8}{12} = 4.17 \times 10^6 = 4.17 \text{ MHz},$$

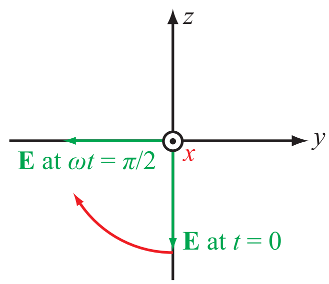
and

$$\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{377}{\sqrt{36}} = 62.83\ \Omega.$$

From Eq. (7.39b),

$$\begin{aligned}
\widetilde{\mathbf{E}} &= -\eta \hat{\mathbf{k}} \times \widetilde{\mathbf{H}} \\
&= -62.83\hat{\mathbf{x}} \times [30(\hat{\mathbf{y}} + j\hat{\mathbf{z}})e^{-j\pi x/6}] \times 10^{-3} \\
&= 1.885(-\hat{\mathbf{z}} + \hat{\mathbf{y}}e^{j\pi/2})e^{-j\pi x/6} \quad \text{(V/m)}, \\
\mathbf{E}(x,t) &= \mathfrak{Re}[\widetilde{\mathbf{E}}e^{j\omega t}] \\
&= 1.885\left[-\hat{\mathbf{y}}\sin\left(\omega t - \frac{\pi x}{6}\right) - \hat{\mathbf{z}}\cos\left(\omega t - \frac{\pi x}{6}\right)\right] \quad \text{(V/m)}, \\
|\mathbf{E}| &= [E_y^2 + E_z^2]^{1/2} = 1.885 \text{ V/m}.
\end{aligned}$$

This is a LHC wave.



At $x = 0$ and $t = 0$, $\mathbf{E} = -\hat{\mathbf{z}}1.885$ (V/m).
At $x = 0$ and $t = 5$ ns,

$$\omega t = 2\pi \times 4.17 \times 10^6 \times 5 \times 10^{-9} = 0.13 \text{ rad},$$

$$\begin{aligned}\mathbf{E} &= -1.885(\hat{\mathbf{y}} \sin 0.13 + \hat{\mathbf{z}} \cos 0.13) \\ &= -1.885(\hat{\mathbf{y}}0.13 + \hat{\mathbf{z}}0.99) \quad \text{(V/m)}.\end{aligned}$$