7.13 A linearly polarized plane wave of the form $\widetilde{\mathbf{E}} = \hat{\mathbf{x}} a_x e^{-jkz}$ can be expressed as the sum of an RHC polarized wave with magnitude a_{R}, and an LHC polarized wave with magnitude a_{L}. Prove this statement by finding expressions for a_{R} and a_{L} in terms of a_x.

Solution:

$$\widetilde{\mathbf{E}} = \hat{\mathbf{x}} a_x e^{-jkz},$$

$$\text{RHC wave:} \quad \widetilde{\mathbf{E}}_{\mathrm{R}} = a_{\mathrm{R}}(\hat{\mathbf{x}} + \hat{\mathbf{y}} e^{-j\pi/2}) e^{-jkz} = a_{\mathrm{R}}(\hat{\mathbf{x}} - j\hat{\mathbf{y}}) e^{-jkz},$$

$$\text{LHC wave:} \quad \widetilde{\mathbf{E}}_{\mathrm{L}} = a_{\mathrm{L}}(\hat{\mathbf{x}} + \hat{\mathbf{y}} e^{j\pi/2}) e^{-jkz} = a_{\mathrm{L}}(\hat{\mathbf{x}} + j\hat{\mathbf{y}}) e^{-jkz},$$

$$\widetilde{\mathbf{E}} = \widetilde{\mathbf{E}}_{\mathrm{R}} + \widetilde{\mathbf{E}}_{\mathrm{L}},$$

$$\hat{\mathbf{x}} a_x = a_{\mathrm{R}}(\hat{\mathbf{x}} - j\hat{\mathbf{y}}) + a_{\mathrm{L}}(\hat{\mathbf{x}} + j\hat{\mathbf{y}}).$$

By equating real and imaginary parts, $a_x = a_{\mathrm{R}} + a_{\mathrm{L}}$, $0 = -a_{\mathrm{R}} + a_{\mathrm{L}}$, or $a_{\mathrm{L}} = a_x/2$, $a_{\mathrm{R}} = a_x/2$.