7.14 The electric field of an elliptically polarized plane wave is given by

$$\mathbf{E}(z,t) = [-\hat{\mathbf{x}}\,10\sin(\omega t - kz - 60^\circ) + \hat{\mathbf{y}}\,30\cos(\omega t - kz)] \qquad \text{(V/m)}$$

Determine the following:

(a) The polarization angles (γ, χ).

(b) The direction of rotation.

Solution:

(a)

$$\begin{aligned}\mathbf{E}(z,t) &= [-\hat{\mathbf{x}}10\sin(\omega t - kz - 60^\circ) + \hat{\mathbf{y}}30\cos(\omega t - kz)]\\ &= [\hat{\mathbf{x}}10\cos(\omega t - kz + 30^\circ) + \hat{\mathbf{y}}30\cos(\omega t - kz)] \qquad \text{(V/m)}.\end{aligned}$$

Phasor form:

$$\widetilde{\mathbf{E}} = (\hat{\mathbf{x}}10e^{j30^\circ} + \hat{\mathbf{y}}30)e^{-jkz}.$$

Since δ is defined as the phase of E_y relative to that of E_x,

$$\begin{aligned}\delta &= -30^\circ,\\ \psi_0 &= \tan^{-1}\left(\frac{30}{10}\right) = 71.56^\circ,\\ \tan 2\gamma &= (\tan 2\psi_0)\cos\delta = -0.65 \quad \text{or} \quad \gamma = 73.5^\circ,\\ \sin 2\chi &= (\sin 2\psi_0)\sin\delta = -0.40 \quad \text{or} \quad \chi = -8.73^\circ.\end{aligned}$$

(b) Since $\chi < 0$, the wave is right-hand elliptically polarized.