

7.22 Ignoring reflection at the air-water boundary, if the amplitude of a 1 GHz incident wave in air is 20 V/m at the water surface, at what depth will it be down to 1 μ V/m? Water has $\mu_r = 1$ and at 1 GHz, $\epsilon_r = 80$ and $\sigma = 1$ S/m.

Solution: For water at 1 GHz,

$$\frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2\pi f\epsilon_r\epsilon_0} = \frac{1}{2\pi \times 10^9 \times 80 \times 8.85 \times 10^{-12}} = 0.225.$$

Hence, it is neither a low-loss nor a conducting medium. Using Eq. (7.66a):

$$\begin{aligned}\alpha &= \omega \left\{ \frac{\mu\epsilon'}{2} \left[\sqrt{1 + \left(\frac{\epsilon''}{\epsilon'} \right)^2} - 1 \right] \right\}^{1/2} \\ &= \omega \left\{ \frac{\mu_r\epsilon_r'}{2c^2} \left[\sqrt{1 + \left(\frac{\epsilon''}{\epsilon'} \right)^2} - 1 \right] \right\}^{1/2} \\ &= 2\pi \times 10^9 \left\{ \frac{80}{2(3 \times 10^8)^2} \left[\sqrt{1 + (0.225)^2} - 1 \right] \right\}^{1/2} \\ &= 20.94 \text{ Np/m.}\end{aligned}$$

The depth d at which E is down to 1 μ V/m is obtained from

$$10^{-6} = 20e^{-\alpha d} = 20e^{-20.94d},$$

which leads to

$$\ln \left(\frac{10^{-6}}{20} \right) = -20.94d,$$

or

$$d = \frac{16.8}{20.94} = 0.804 \text{ m} = 80.4 \text{ cm}.$$
