7.35 The electric-field phasor of a uniform plane wave traveling downward in water is given by

$$\widetilde{\mathbf{E}} = \hat{\mathbf{x}}\, 5e^{-0.2z}e^{-j0.2z} \qquad \text{(V/m)}$$

where $\hat{\mathbf{z}}$ is the downward direction and $z = 0$ is the water surface. If $\sigma = 4$ S/m,

(a) Obtain an expression for the average power density.

(b) Determine the attenuation rate.

(c) Determine the depth at which the power density has been reduced by 40 dB.

Solution:

(a) Since $\alpha = \beta = 0.2$, the medium is a good conductor.

$$\eta_c = (1+j)\frac{\alpha}{\sigma} = (1+j)\frac{0.2}{4} = (1+j)0.05 = 0.0707e^{j45^\circ} \qquad (\Omega).$$

From Eq. (7.109),

$$\mathbf{S}_{\text{av}} = \hat{\mathbf{z}}\frac{|E_0|^2}{2|\eta_c|}e^{-2\alpha z}\cos\theta_\eta = \hat{\mathbf{z}}\frac{25}{2\times 0.0707}e^{-0.4z}\cos 45^\circ = \hat{\mathbf{z}}125e^{-0.4z} \qquad (\text{W/m}^2).$$

(b) $A = -8.68\alpha z = -8.68\times 0.2z = -1.74z$ (dB).

(c) 40 dB is equivalent to 10^{-4}. Hence,

$$10^{-4} = e^{-2\alpha z} = e^{-0.4z}, \qquad \ln(10^{-4}) = -0.4z,$$

or $z = 23.03$ m.