

**7.40** Repeat Problem 7.39 for a wave traveling in a lossy medium in which

$$\mathbf{E} = \hat{\mathbf{x}} 100e^{-20y} \cos(2\pi \times 10^9 t - 40y) \quad (\text{V/m})$$

$$\mathbf{H} = -\hat{\mathbf{z}} 0.64e^{-20y} \cos(2\pi \times 10^9 t - 40y - 36.85^\circ) \quad (\text{A/m})$$

The box has dimensions  $a = 1$  cm,  $b = 2$  cm, and  $c = 0.5$  cm.

**Solution:**

(a)

$$\begin{aligned} \mathbf{S}(t) &= \mathbf{E} \times \mathbf{H} \\ &= \hat{\mathbf{x}} 100e^{-20y} \cos(2\pi \times 10^9 t - 40y) \\ &\quad \times (-\hat{\mathbf{z}} 0.64)e^{-20y} \cos(2\pi \times 10^9 t - 40y - 36.85^\circ) \\ &= \hat{\mathbf{y}} 64e^{-40y} \cos(2\pi \times 10^9 t - 40y) \cos(2\pi \times 10^9 t - 40y - 36.85^\circ). \end{aligned}$$

Using the identity  $\cos \theta \cos \phi = \frac{1}{2}[\cos(\theta + \phi) + \cos(\theta - \phi)]$ ,

$$\begin{aligned} S(t) &= \frac{64}{2} e^{-40y} [\cos(4\pi \times 10^9 t - 80y - 36.85^\circ) + \cos 36.85^\circ], \\ P(t) &= S(t) A|_{y=0} - S(t) A|_{y=b} \\ &= 32ac \{ [\cos(4\pi \times 10^9 t - 36.85^\circ) + \cos 36.85^\circ] \\ &\quad - e^{-40b} [\cos(4\pi \times 10^9 t - 80y - 36.85^\circ) + \cos 36.85^\circ] \}. \end{aligned}$$

(b)

$$P_{\text{av}} = \frac{1}{T} \int_0^T P(t) dt = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} P(t) dt.$$

The average of  $\cos(\omega t + \theta)$  over a period  $T$  is equal to zero, regardless of the value of  $\theta$ . Hence,

$$P_{\text{av}} = 32ac(1 - e^{-40b}) \cos 36.85^\circ.$$

With  $a = 1$  cm,  $b = 2$  cm, and  $c = 0.5$  cm,

$$P_{\text{av}} = 7.05 \times 10^{-4} \quad (\text{W}).$$

This is the average power absorbed by the lossy material in the box.

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