7.42 A team of scientists is designing a radar as a probe for measuring the depth of the ice layer over the antarctic land mass. In order to measure a detectable echo due to the reflection by the ice-rock boundary, the thickness of the ice sheet should not exceed three skin depths. If $\varepsilon_r' = 3$ and $\varepsilon_r'' = 10^{-2}$ for ice and if the maximum anticipated ice thickness in the area under exploration is 1.2 km, what frequency range is useable with the radar?

Solution:

$$3\delta_s = 1.2 \text{ km} = 1200 \text{ m}$$
$$\delta_s = 400 \text{ m}.$$

Hence,

$$\alpha = \frac{1}{\delta_s} = \frac{1}{400} = 2.5 \times 10^{-3} \qquad \text{(Np/m)}.$$

Since $\varepsilon''/\varepsilon' \ll 1$, we can use (7.75a) for α:

$$\alpha = \frac{\omega\varepsilon''}{2}\sqrt{\frac{\mu}{\varepsilon'}} = \frac{2\pi f \varepsilon_r'' \varepsilon_0}{2\sqrt{\varepsilon_r'}\sqrt{\varepsilon_0}}\sqrt{\mu_0} = \frac{\pi f \varepsilon_r''}{c\sqrt{\varepsilon_r}} = \frac{\pi f \times 10^{-2}}{3 \times 10^8 \sqrt{3}} = 6f \times 10^{-11} \text{Np/m}.$$

For $\alpha = 2.5 \times 10^{-3} = 6f \times 10^{-11}$,

$$f = 41.6 \text{ MHz}.$$

Since α increases with increasing frequency, the useable frequency range is

$$f \leq 41.6 \text{ MHz}.$$