

8.2 A plane wave traveling in medium 1 with $\epsilon_{r1} = 2.25$ is normally incident upon medium 2 with $\epsilon_{r2} = 4$. Both media are made of nonmagnetic, non-conducting materials. If the electric field of the incident wave is given by

$$\mathbf{E}^i = \hat{\mathbf{y}} 8 \cos(6\pi \times 10^9 t - 30\pi x) \quad (\text{V/m}).$$

- (a) Obtain time-domain expressions for the electric and magnetic fields in each of the two media.
- (b) Determine the average power densities of the incident, reflected and transmitted waves.

Solution:

(a)

$$\mathbf{E}^i = \hat{\mathbf{y}} 8 \cos(6\pi \times 10^9 t - 30\pi x) \quad (\text{V/m}),$$

$$\eta_1 = \frac{\eta_0}{\sqrt{\epsilon_{r1}}} = \frac{\eta_0}{\sqrt{2.25}} = \frac{\eta_0}{1.5} = \frac{377}{1.5} = 251.33 \, \Omega,$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\epsilon_{r2}}} = \frac{\eta_0}{\sqrt{4}} = \frac{377}{2} = 188.5 \, \Omega,$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{1/2 - 1/1.5}{1/2 + 1/1.5} = -0.143,$$

$$\tau = 1 + \Gamma = 1 - 0.143 = 0.857,$$

$$\mathbf{E}^r = \Gamma \mathbf{E}^i = -1.14 \hat{\mathbf{y}} \cos(6\pi \times 10^9 t + 30\pi x) \quad (\text{V/m}).$$

Note that the coefficient of x is positive, denoting the fact that \mathbf{E}^r belongs to a wave traveling in $-x$ -direction.

$$\mathbf{E}_1 = \mathbf{E}^i + \mathbf{E}^r = \hat{\mathbf{y}} [8 \cos(6\pi \times 10^9 t - 30\pi x) - 1.14 \cos(6\pi \times 10^9 t + 30\pi x)] \quad (\text{A/m}),$$

$$\mathbf{H}^i = \hat{\mathbf{z}} \frac{8}{\eta_1} \cos(6\pi \times 10^9 t - 30\pi x) = \hat{\mathbf{z}} 31.83 \cos(6\pi \times 10^9 t - 30\pi x) \quad (\text{mA/m}),$$

$$\mathbf{H}^r = \hat{\mathbf{z}} \frac{1.14}{\eta_1} \cos(6\pi \times 10^9 t + 30\pi x) = \hat{\mathbf{z}} 4.54 \cos(6\pi \times 10^9 t + 30\pi x) \quad (\text{mA/m}),$$

$$\mathbf{H}_1 = \mathbf{H}^i + \mathbf{H}^r$$

$$= \hat{\mathbf{z}} [31.83 \cos(6\pi \times 10^9 t - 30\pi x) + 4.54 \cos(6\pi \times 10^9 t + 30\pi x)] \quad (\text{mA/m}).$$

Since $k_1 = \omega \sqrt{\mu \epsilon_1}$ and $k_2 = \omega \sqrt{\mu \epsilon_2}$,

$$k_2 = \sqrt{\frac{\epsilon_2}{\epsilon_1}} k_1 = \sqrt{\frac{4}{2.25}} 30\pi = 40\pi \quad (\text{rad/m}),$$

$$\mathbf{E}_2 = \mathbf{E}^t = \hat{\mathbf{y}} 8\tau \cos(6\pi \times 10^9 t - 40\pi x) = \hat{\mathbf{y}} 6.86 \cos(6\pi \times 10^9 t - 40\pi x) \quad (\text{V/m}),$$

$$\mathbf{H}_2 = \mathbf{H}^t = \hat{\mathbf{z}} \frac{8\tau}{\eta_2} \cos(6\pi \times 10^9 t - 40\pi x) = \hat{\mathbf{z}} 36.38 \cos(6\pi \times 10^9 t - 40\pi x) \quad (\text{mA/m}).$$

(b)

$$\mathbf{S}_{\text{av}}^i = \hat{\mathbf{x}} \frac{8^2}{2\eta_1} = \frac{64}{2 \times 251.33} = \hat{\mathbf{x}} 127.3 \quad (\text{mW/m}^2),$$

$$\mathbf{S}_{\text{av}}^r = -|\Gamma|^2 \mathbf{S}_{\text{av}}^i = -\hat{\mathbf{x}} (0.143)^2 \times 0.127 = -\hat{\mathbf{x}} 2.6 \quad (\text{mW/m}^2),$$

$$\begin{aligned} \mathbf{S}_{\text{av}}^t &= \frac{|E_0^t|^2}{2\eta_2} \\ &= \hat{\mathbf{x}} \tau^2 \frac{(8)^2}{2\eta_2} = \hat{\mathbf{x}} \frac{(0.86)^2 64}{2 \times 188.5} = \hat{\mathbf{x}} 124.7 \quad (\text{mW/m}^2). \end{aligned}$$

Within calculation error, $\mathbf{S}_{\text{av}}^i + \mathbf{S}_{\text{av}}^r = \mathbf{S}_{\text{av}}^t$.
