8.29 A plane wave in air with

$$\widetilde{\mathbf{E}}^{\mathrm{i}} = (\hat{\mathbf{x}}\,9 - \hat{\mathbf{y}}\,4 - \hat{\mathbf{z}}\,6)e^{-j(2x+3z)} \qquad \text{(V/m)}$$

is incident upon the planar surface of a dielectric material, with $\varepsilon_{\mathrm{r}} = 2.25$, occupying the half-space $z \geq 0$. Determine

(a) The incidence angle θ_{i}.

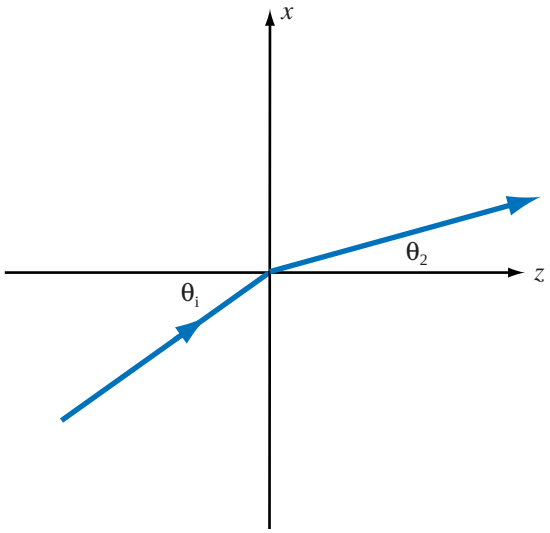
(b) The frequency of the wave.

(c) The field $\widetilde{\mathbf{E}}^{\mathrm{r}}$ of the reflected wave.

(d) The field $\widetilde{\mathbf{E}}^{\mathrm{t}}$ of the wave transmitted into the dielectric medium.

(e) The average power density carried by the wave into the dielectric medium.

Solution:



(a) From the exponential of the given expression, it is clear that the wave direction of travel is in the x–z plane. By comparison with the expressions in (8.48a) for perpendicular polarization or (8.65a) for parallel polarization, both of which have the same phase factor, we conclude that:

$$k_1 \sin\theta_{\mathrm{i}} = 2,$$
$$k_1 \cos\theta_{\mathrm{i}} = 3.$$

Hence,

$$k_1 = \sqrt{2^2 + 3^2} = 3.6 \qquad \text{(rad/m)}$$

$$= (\hat{\mathbf{x}}\,7.65 - \hat{\mathbf{y}}\,3 - \hat{\mathbf{z}}\,3.05)e^{-j(2x+5z)} \qquad \text{(V/m)}.$$

(e)

$$S^{\mathrm{t}} = \frac{|E_0^{\mathrm{t}}|^2}{2\eta_2}$$

$$|E_0^{\mathrm{t}}|^2 = (7.65)^2 + 3^2 + (3.05)^2 = 76.83$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{\mathrm{r}_2}}} = \frac{377}{1.5} = 251.3\ \Omega$$

$$S^{\mathrm{t}} = \frac{76.83}{2 \times 251.3} = 152.86 \qquad (\mathrm{mW/m^2}).$$